3E1 Problem Sheet 3

October 27 - November 2, 2003

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1. Let f be the 2π-periodic function determined by

$$f(x) = x^2 \text{ for } -\pi \leq x \leq \pi \text{ and } f(x+2\pi) = f(x), \; x \in \mathbb{R}.$$

   (a) Find the complex Fourier coefficients of f.

   (b) Use (a) to obtain the real Fourier series of f.

   (c) Give a reason why the complex Fourier coefficients computed in (a) are real numbers, rather than complex numbers.

2. *Parseval's identity* states that

$$2a_0 + \sum_{n=1}^{\infty}(a_n^2 + b_n^2) = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)^2 dx$$

   for every 2π-periodic function f with (real) Fourier coefficients a_n ($n \geq 0$) and b_n ($n \geq 1$) and such that the integral on the right exists. Use this to show that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

3. Give a formula for the complex Fourier coefficients of a $2L$-periodic function. Give reasons for your answer.