3E1 Problem Sheet 4

November 3 - 9, 2003

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1. Recall that the representation of a function $f(x)$ by a Fourier integral is of the form

$$f(x) = \int_0^\infty A(w) \cos wx + B(w) \sin wx dw.$$

   (a) State the formulae for $A(w)$ and $B(w)$.

   (b) Find the Fourier integral representation of the function

$$f(x) = \begin{cases} 1, & -1 < x < 1 \\ 0, & |x| > 1 \end{cases} .$$

   (c) Can you also represent the function $f(x) = k$, $0 \neq k$ a constant, as a Fourier integral? Justify your answer!

2. Using the thoerem from last Wednesday's lecture about Fourier integrals and Exercise 1(b), evaluate the following integral:

$$\int_0^\infty \frac{\cos wx \sin w}{w} dw.$$

   Hint: your answer should depend on x and there are three cases!

3. Use the above method to show that

$$\int_0^\infty \frac{1 - \cos \pi w}{w} \sin xw \, dw = \begin{cases} \frac{\pi}{2}, & \text{if } 0 < x < \pi \\ \frac{\pi}{4}, & x = \pi \\ 0, & x > \pi \end{cases} .$$