3E1 Problem Sheet 5

November 10 - 16, 2003

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1. Sketch the graphs and find the Fourier transforms of the following functions.

   (a)
   $$f(x) = \begin{cases} e^{-ax}, & \text{if } x > 0 \\ 0, & \text{if } x < 0 \end{cases} \qquad \text{with } a > 0 \text{, a constant.}$$

   (b)
   $$f(x) = \begin{cases} e^{bx}, & \text{if } x < 0 \\ 0, & \text{if } x > 0 \end{cases} \qquad \text{with } b > 0 \text{, a constant.}$$

   (c)
   $$f(x) = \begin{cases} ce^{ax}, & \text{if } x < 0 \\ ce^{-ax}, & \text{if } x > 0 \end{cases} \qquad \text{where } c \neq 0 \text{ and } a > 0 \text{ are constants.}$$

2. The formulae in parts (a) and (b) below are called **Shift Theorems**.

   (a) Suppose $f(x)$ is function which has a Fourier transform. Argue that $f(x-c)$, c a constant, also has a Fourier transform. Then show that
   $$\mathcal{F}(f(x-c))(w) = e^{-icw}\mathcal{F}(f(x))(w).$$

   (b) Show that if $\hat{f}(w)$ is the Fourier transform of $f(x)$, then $\hat{f}(w-c)$ is the Fourier transform of $e^{icx}f(x)$.

3. Recal from the lecture that the Fourier transform of
   $$f(x) = \begin{cases} 1, & -a < x < a \\ 0, & |x| > a \end{cases} \qquad \text{is} \qquad \hat{f}(w) = \sqrt{\frac{2}{\pi}}\,\frac{\sin aw}{w}.$$

   Use this and a shift theorem to obtain the Fourier transform of

   (a)
   $$g(x) = \begin{cases} e^{icx}, & -a < x < a \\ 0, & \text{otherwise} \end{cases}.$$

   (b)
   $$h(x) = \begin{cases} 3, & b < x < c \\ 0, & x < b \text{ or } x > c \end{cases}, \quad \text{where } b < c \text{ are constants.}$$