3E1 Problem Sheet 6*

November 17 - 23, 2003

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1. Recall from the lectures that the method of separation of variables, i.e. setting $u(x,t) = F(x)G(t)$, applied to the one-dimensional heat (or diffusion) equation
$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \tag{HE}$$
lead to the following two ordinary differential equations
$$\frac{\partial^2 F}{\partial x^2} - kF = 0 \quad \text{and} \quad \frac{\partial G}{\partial t} - kc^2 G = 0,$$
where k is the so called *separation constant.*
   (a) Below L denotes the length of the wire under consideration, so $L > 0$. Show that the boundary conditions
   $$u(0,t) = 0 \quad \text{and} \quad u(L,t) = 0 \tag{BC}$$
   together with the assumption $k \geq 0$ imply that $u(x,t) = 0$.
   (b) Why do you need to consider the cases $k = 0$ and $k > 0$ separately in part (a)?
   (c) Say in words what the boundary conditions (BC) mean.

2. In the lecture we found the general solution of (HE) satisfying (BC) to be
$$u(x,t) = \sum_{n=1}^{\infty} B_n \left(\sin \frac{n\pi x}{L}\right) e^{-\lambda_n^2 t}, \qquad \text{where } \lambda_n = \frac{cn\pi}{L},\ n = 1, 2, 3, \ldots$$
(Here $c^2 = \frac{K}{\sigma\rho}$ is a constant depending on the material of the wire.)
   (a) Find a solution of (HE) satisfying (BC) and the initial condition
   $$u(x,0) = 8x(L-x) \quad \text{for} \quad 0 \leq x \leq L. \tag{IC}$$
   (b) Give a reason why $u(x,0)$ from part (a) is a valid initial condition and sketch its graph.

3. (a) Decide whether (HE) is an elliptic, parabolic or hyperbolic PDE.†
   (b) State the one-dimensional wave equation and decide whether it is an elliptic, parabolic or hyperbolic PDE.‡

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*In general, it pays off to first read through all problems, as some are easier than others.
†PDE stands for partial differential equation.
‡Not reading footnotes can, in real life, be very expensive! Here were two easy marks.