

## 3E1 Problem Sheet 9

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1. The *complex conjugate of the complex number*  $z = x + iy$ , which is usually denoted by  $\bar{z}$ , is defined by  $\bar{z} = x - iy$ .
  - (a) Let  $z_1 = 3 - 2i$  and  $z_2 = 4 + i$ . In the complex plane, draw the following complex numbers and their complex conjugates:  $z_1$ ,  $z_2$  and  $z_1 + z_2$ .
  - (b) Show that  $\operatorname{Re}(z) = \frac{1}{2}(z + \bar{z})$  and  $\operatorname{Im}(z) = \frac{-i}{2}(z - \bar{z})$  hold true for every complex number  $z = x + iy$ .
  - (c) Recall that the *absolute value* (also called *modulus*) of  $z = x + iy$ , denoted  $|z|$ , is defined by  $|z| = \sqrt{x^2 + y^2}$ . Prove the following three identities:  $z\bar{z} = |z|^2$ ,  $\overline{z + w} = \bar{z} + \bar{w}$  and  $\overline{zw} = \bar{z}\bar{w}$ .
2. Let  $f(z) = \frac{1}{z}$  and write  $z = x + iy$ .
  - (a) Find real functions  $u(x, y)$  and  $v(x, y)$  such that  $f(z) = u(x, y) + iv(x, y)$ . (Hint: multiply  $f(z)$  by  $\frac{\bar{z}}{\bar{z}}$ .)
  - (b) Show that the functions  $u$  and  $v$  obtained in part (a) satisfy the Cauchy-Riemann equations.
  - (c) Show that  $f(z)$  is differentiable everywhere except at 0 and find its derivative.
3. Decide whether there are any points  $z_0$  at which  $f(z) = \bar{z}$  is differentiable, where  $\bar{z}$  denotes the complex conjugate of  $z$ . Justify your answer.