1S1 Problem Sheet 1

October 18–31, 2004

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QUESTION 1. Prove the following statement using only the three basic rules of addition and multiplication, i.e. commutativity, associativity and distributivity, where a^2 is defined by $a^2 = aa$. Justify each step!

$$\forall a, b \in \mathbb{R} \quad (a+b)^2 = a^2 + 2ab + b^2$$

QUESTION 2. Give a general, not recursive, formula of the form $a_n = \ldots$ for each of the following sequences.

(a) $(1, 3, 5, 7, 9, \ldots)$ (b) $(2, 4, 6, 8, 10, \ldots)$
(c) $(-1, 1, -1, 1, -1, 1, \ldots)$ (d) $(-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}, \ldots)$

QUESTION 3. For each of the following recursively defined sequences write down the first six terms.

(a) $a_1 = 2,\ a_{n+1} = 3a_n$ (b) $a_1 = 1,\ a_2 = 2,\ a_{n+2} = a_{n+1} - a_n$
(c) $a_1 = 1,\ a_{n+1} = \frac{n}{a_n}$ (d) $a_1 = 1,\ a_{n+1} = a_n + n + 1$

QUESTION 4. Consider the following definitions.

Definition 1. A sequence $(a_1, a_2, a_3, a_4, \ldots)$ is periodic of period p if $p \in \mathbb{N}$ and $a_{n+p} = a_n$ holds true for all $n \in \mathbb{N}$.

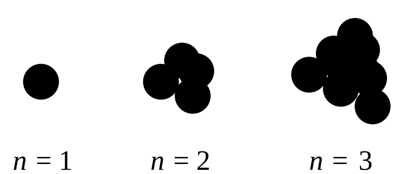
Definition 2. A sequence is called periodic if it is periodic of period p for some $p \in \mathbb{N}$.

a) Convince yourself that the sequence in Question 2(c) above is periodic and give a value for p such that it is periodic of period p.

b) Show that the sequence in Question 3(b) is periodic of period 6.

c) Prove that a sequence which is periodic of period p is also periodic of period mp for all $m \in \mathbb{N}$.

QUESTION 5. Let a_n be the total number of cannon balls in a pyramid shaped pile of cannon balls whose base is an equelateral triangle with n balls along each side (see below). Find and justify a possibly recursive formula for a_n.



QUESTION 6. Prove the statement in Question 1 geometrically.

QUESTION 7. Find, state and justify a relationship between the sequences in Questions 5 and 3(d).