1S1 Problem Sheet 2

November 1, 2004, Lecturer: Claas Röver

Solutions to **three** questions are due **Thursday, 11 Nov.** before the lecture

Recall that for every real number a, $a^0 = 1$ and $a^k = a \cdot a^{k-1}$ for $k \in \mathbb{N}$.

QUESTION 1. Prove the following statements by induction.

(a) For $x \neq 1$ and every $n \in \mathbb{N}$ the following is true

$$\sum_{k=0}^{n} x^k = \frac{1 - x^{n+1}}{1 - x}.$$

(b) For all $n \in \mathbb{N}$

$$\sum_{k=1}^{n} k^2 = \frac{1}{6} n(n+1)(2n+1) \text{ holds.}$$

(c) The inequality $n^2 < 2^n$ holds for all $n \geq 5$.

QUESTION 2. For each of the following sequences, determine whether they converge, and if so find their limits.

(a) $a_n = \dfrac{2n^2 + 5n - 2}{n^2 - 103n}$, $n \in \mathbb{N}$

(b) $a_n = \dfrac{n^3}{2^n}$, $n \in \mathbb{N}$

(c) $a_n = (-1)^n \dfrac{3n^2}{n^2 - 1}$, $n \in \mathbb{N}$

(d) $a_n = \begin{cases} 0, & \text{if } n \text{ is odd} \\ \frac{2}{n}, & \text{if } n \text{ is even} \end{cases}$ $n \in \mathbb{N}$

QUESTION 3. Find the sums of the following serieses.

(a) $\displaystyle\sum_{k=1}^{\infty} 4 \left(\frac{1}{10}\right)^k$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{2^k}{3^{k-1}}$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$

QUESTION 4. Find the domain and range of the following functions and sketch their graphs.

(a) $f(x) = (x-1)^2$

(b) $g(x) = \dfrac{1}{x}$

(c) $h(x) = x^3$

QUESTION 5. A drink can in the shape of a cylindrical box has volume $V = \frac{1}{3} l$.

(a) Find a function S which describes the surface area of the can in dependence on the radius r of the base (or top) of the can.

(b) Determine the domain and range of the function $S(r)$ you found in part (a).

(c) Sketch the graph of the function $S(r)$.