1S1 Problem Sheet 3

November 15, 2004, Lecturer: Claas Röver

Solutions to **three** questions are due **Thursday, 25 November** before the lecture. Give your NAME and GROUP NUMBER on the solutions and STAPLE them.

QUESTION 1. For each of the following functions, determine whether

$$\lim_{x\to 0} f(x),\ \lim_{x\to 1} f(x),\ \lim_{x\to\infty} f(x) \text{ and } \lim_{x\to-\infty} f(x) \text{ exist, and if so find their values.}$$

$$(a)\quad f(x)=\frac{x^2+3x-4}{x-1} \qquad (b)\quad g(x)=\frac{1}{x^2} \qquad (c)\quad h(x)=\begin{cases} 0, & x\le 0 \\ x, & x>0 \end{cases}$$

QUESTION 2. (a) Draw, in one diagram and as accurate as you can, the graphs of the functions $f(x)=3x-2$ and $g(x)=-\frac{1}{2}x+3$.

(b) Read off the x-coordinate of the point P where the two graphs intersect.

(c) Determine the x-coordinate of P by solving $f(x)=g(x)$ for x.

(d) Find the equation for the line l_0 which goes through the origin and P.

(e) Find the equation for the line l_1 which goes through P and intersects the y-axis at height 1.

(f) What is the area of the triangle enclosed by the lines l_0, l_1 and the y-axis?

QUESTION 3. For $n\in\mathbb{N}$ define S_n to be the set of all those finite sequences $(a_1,a_2,\ldots,a_m)$, $m\in\mathbb{N}$, whose terms are either 1 or 2 and so that the sum of all term in the sequence equals n. For example, $S_1=\{(1)\}$ and $S_4=\{(1,1,1,1),(1,1,2),(1,2,1),(2,1,1),(2,2)\}$. Remember that the order of the terms in a sequence is important. Let a_n denote the number of elements of the set S_n, e.g. $a_1=1$ and $a_4=5$. Find and justify a recursive definition of a_n.

QUESTION 4. Let f,g and h be the functions defined in Question 1.

(a) For each of the functions f,g,h find their domain and decide whether they are continuous. Justify your answers.

(b) Write down explicite and simple formulae for the functions $f\circ g$, $h\circ g$ and $f\circ h$, and find their zeros.

(c) Find the zeros of $p(x)=4x^4-5x^2+1$. Hint: Set $z=x^2$.

QUESTION 5. Determine the derivatives of the following functions.

$$(a)\quad r(x)=(x^2+5)^2-6x \qquad (b)\quad s(x)=\sqrt{4x}$$

$$(c)\quad t(x)=\frac{3x^2-4x+1}{x+1} \qquad (d)\quad u(x)=\frac{2}{3}x\sqrt{2-x}$$

QUESTION 6. You are given 24 meters of electric fence wire and three fence posts. Then you are told to fence in an area in the shape of an isosceles[1] triangle. What is the largest area you can fence in?

[1] An isosceles triangle is a triangle in which two sides have the same length.