1S1 Problem Sheet 3

November 15, 2004, Lecturer: Claas Röver

Solutions to **three** questions are due **Thursday, 25 November** before the lecture. Give your NAME and GROUP NUMBER on the solutions and STAPLE them.

QUESTION 1. For each of the following functions, determine whether

$$\lim_{x\to 0} f(x),\ \lim_{x\to 1} f(x),\ \lim_{x\to\infty} f(x) \text{ and } \lim_{x\to-\infty} f(x) \text{ exist, and if so find their values.}$$

$$(a)\quad f(x) = \frac{x^2+3x-4}{x-1} \qquad (b)\quad g(x) = \frac{1}{x^2} \qquad (c)\quad h(x) = \begin{cases} 0, & x \le 0 \\ x, & x > 0 \end{cases}$$

QUESTION 2. (a) Draw, in one diagram and as accurate as you can, the graphs of the functions $f(x) = 3x - 2$ and $g(x) = -\frac{1}{2}x + 3$.

(b) Read off the x-coordinate of the point P where the two graphs intersect.

(c) Determine the x-coordinate of P by solving $f(x) = g(x)$ for x.

(d) Find the equation for the line l_0 which goes through the origin and P.

(e) Find the equation for the line l_1 which goes through P and intersects the y-axis at height 1.

(f) What is the area of the triangle enclosed by the lines l_0, l_1 and the y-axis?

QUESTION 3. For $n \in \mathbb{N}$ define S_n to be the set of all those finite sequences $(a_1, a_2, \ldots, a_m)$, $m \in \mathbb{N}$, whose terms are either 1 or 2 and so that the sum of all term in the sequence equals n. For example, $S_1 = \{(1)\}$ and $S_4 = \{(1,1,1,1), (1,1,2), (1,2,1), (2,1,1), (2,2)\}$. Remember that the order of the terms in a sequence is important. Let a_n denote the number of elements of the set S_n, e.g. $a_1 = 1$ and $a_4 = 5$. Find and justify a recursive definition of a_n.

QUESTION 4. Let f, g and h be the functions defined in Question 1.

(a) For each of the functions f, g, h find their domain and decide whether they are continuous. Justify your answers.

(b) Write down explicite and simple formulae for the functions $f \circ g$, $h \circ g$ and $f \circ h$, and find their zeros.

(c) Find the zeros of $p(x) = 4x^4 - 5x^2 + 1$. Hint: Set $z = x^2$.

QUESTION 5. Determine the derivatives of the following functions.

$$(a)\quad r(x) = (x^2+5)^2 - 6x \qquad (b)\quad s(x) = \sqrt{4x}$$

$$(c)\quad t(x) = \frac{3x^2 - 4x + 1}{x+1} \qquad (d)\quad u(x) = \frac{2}{3}x\sqrt{2-x}$$

QUESTION 6. You are given 24 meters of electric fence wire and three fence posts. Then you are told to fence in an area in the shape of an isosceles[1] triangle. What is the largest area you can fence in?

[1] An <u>isosceles</u> triangle is a triangle in which two sides have the same length.