1S1 Problem Sheet 4

November 29, 2004, Lecturer: Claas Röver

Solutions to **three** questions are due **Thursday, 9 December** before the lecture. Give your NAME and GROUP NUMBER on the solutions and STAPLE them.

QUESTION 1. Determine the domain, zeros, $\lim_{x\to\infty} f(x)$, $\lim_{x\to-\infty} f(x)$, extrema and points of inflection of the following functions and sketch their graphs.

$$(a)\quad f(x) = x^3 - 3x^2 + 2x \qquad (b)\quad f(x) = \sqrt{x^2 - 4}$$

Then compute $\lim_{x\to\infty} f'(x)$ and $\lim_{x\to-\infty} f'(x)$ in the above cases and interpret your results. Redraw you sketches if these results are not yet reflected in them.

QUESTION 2. A rectangle has its lower corners on the x-axis and its upper corners on the graph of the function f. What are the side lengths and area of the largest such rectangle for the following functions f?

$$(a)\quad f(x) = 18 - x^2 \qquad (b)\quad f(x) = 18 - (x+2)^2 \qquad (c)\quad f(x) = 14 + 4x - x^2$$

Can you explain why you got the same answer in all three cases?

QUESTION 3. (a) You are asked to build a closed, right-angled box of volume 18 cubic meters whose base is a square. You must use two types of wood, ash for the top and bottom which costs 18 Euro per square meter and teak for the four sides which costs 125 Euro per square meter. Find the dimensions and the price of the cheapest such box.

(b) Use the chain rule to find the derivative of $f(x) = \sqrt[4]{x}$.

(c) Find the derivative of $f(x) = \sqrt[3]{x}$ from first principles by using that $a^3 - b^3 = (a-b)(a^2+ab+b^2)$.

QUESTION 4. At time t a train is at the point $(t, f(t))$ in the xy-plane, where $f(t) = 3t^2 + 4t - 6$. (Interpret negative t as past and positive t as future.) Find the times at which the train was/will be closest to the following points.
HINT: Minimise the square of the distance rather than the distance itself.

$$(a)\quad P = (4, -7) \qquad (b)\quad Q = (-3, -8) \qquad (c)\quad R = \left(\frac{-2}{3}, \frac{-35}{6}\right)$$

Can you tell from the results above at which time t_0, the train is at the lowest point of the parabola defined by f?

QUESTION 5. Find the first and second derivatives of the following functions.

$$(a)\quad p(x) = \sin(x^3 - 2x) \qquad (b)\quad q(x) = \sqrt{2x-4}\cos\left(\frac{1}{x}\right)$$

$$(c)\quad r(x) = \frac{\sin(2x)}{\cos(x-4)} \qquad (d)\quad s(x) = \cos^2(x)$$