1S1 Problem Sheet 5

January 10, 2005, Lecturer: Claas Röver

Solutions to **three** questions are due **Thursday, 20 January** before the lecture.
Give your NAME and GROUP NUMBER on the solutions and STAPLE them.

QUESTION 1. Sketch the graphs of the following functions for $-2\pi \leq x \leq 2\pi$ so that at least the zeros and relative extrema are in the right places.

$(i)\quad f_1(x) = \sin x \qquad (ii)\quad f_2(x) = \sin(2x) \qquad (iii)\quad f_3(x) = \sin(3x).$

$(iv)\quad g_1(x) = \cos x \qquad (v)\quad g_2(x) = 2\cos x \qquad (vi)\quad g_3(x) = 3\cos x.$

For a real positive constant k define functions $f_k(x) = \sin(kx)$ and $g_k(x) = k\cos x$. What can you say about the graphs of f_k and g_k?

QUESTION 2. (a) Find an interval of length at most $\frac{1}{10}$ which contains a zero of the function $f(x) = x^3 + 2x + 1$.

(b) Show that the equation $\cos x = \frac{1}{2}x$ has a solution in the interval $[0, 1.2]$.

(c) Apply Newton's method to find an approximation of the zero of $f(x) = x^4 - x - 3$ in the interval $[1, 2]$. Take $x_1 = 1.4$ as your first estimate and calculate $x_2, x_3, \ldots$ until your calculator yields $x_{n+1} = x_n$.

(d) Approximate, as good as your calculator permits, the zero of $f(x) = -x^3 + x^2 - 4$, using Newton's Method. Give each step of the approximation, not just a final answer.

QUESTION 3. (a) Prove that $x \geq \sin x$ for all $x \geq 0$.

(b) Find the zeros, relative extrema and inflection points of the function $f(x) = \sin(x)\cos(x)$ and sketch its graph for $-2\pi \leq x \leq 2\pi$.

(c) Use the Addition Formulae for sine and cosine to prove that
$\cos(2x) = \cos^2(x) - \sin^2(x)$ and $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$.

QUESTION 4. (a) A spherical balloon is inflated so that its volume increases at the constant rate of $4\text{cm}^3/\text{sec}$. How fast is the radius of the balloon increasing at the times when the radius is (i) 10cm and (ii) 6cm?

(b) Now the balloon is deflated air so that its radius changes at the constant rate of 2cm/sec. Determine how fast the volume changes when the radius is 5cm.

(c) Show that, if the balloon is is deflated so that its volume changes at a rate proportional to its surface area, then its radius changes at a constant rate.

QUESTION 5. A point particle P is moving along the curve given by $y = \sqrt{x}$. Another point particle Q is moving a long the curve given by $y = \frac{10x}{x^2+1}$.

(a) Suppose that the x-coordinate of P is increasing at the rate of $\frac{1}{4}$ units/sec when $x = 3$. How fast is the distance between P and the point $(0, 2)$ changing at this instant? Is the distance increasing or decreasing?

(b) Suppose that for $x = 2$ the x-coordinates of P and Q are changing at the rate of 2 respectively 3 units/sec. How fast is the distance between P and Q changing at this instant? Are they getting closer or are they moving away from each other?

(c) Find an approximation, up to 8 decimal places, of a positive x-coordinate at which the two paths of the particles P and Q cross, if there is one.