

2S1 Problem Sheet 1

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QUESTION 1. For the following functions determine their domains, decide where they are continuous and find their partial derivatives.

(a) $f(x, y) = \frac{x^2 y}{x-y}$

(b) $f(x, y) = x^2 \sin y + y \cos x$

(c) $f(x, y) = \ln(xy)$

(d) $f(x, y) = \frac{1}{x} e^{x^2+y^2}$

(e) $f(x, y) = 2|x| - 3|y|$

(f) $f(x, y) = \sqrt{x(x-y)}$

QUESTION 2. In your own words, describe what the partial derivatives of a function of two variables describe.

QUESTION 3. Determine the partial derivatives of the following functions

(a) $f(x, y) = \sin(xy - 3x)$

(b) $f(x, y) = (x - 9y^2)e^{-3x+4y}$.

(c) $f(x, y) = (\ln x) \cos(2x^3)$

(d) $f(x, y) = \frac{2e^{-x^2}}{x+y}$

QUESTION 4. Sketch the level curves of the following functions for four different values of your choice.

(a) $f(x, y) = x^2 + y^2$

(b) $f(x, y) = 3x - y$

(c) $f(x, y) = x^3 + 2y$

(d) $f(x, y) = xe^y$

QUESTION 5. Define $f(x, y) = \frac{x-y}{x^2-y^2}$.

(a) Find the limit of $f(x, y)$ at $(0, 0)$ along the following curves.

(i) The positive x -axis; i.e. $x(t) = t$, $y(t) = 0$, $t > 0$.

(ii) The negative x -axis.

(iii) Any ray emanating from the origin into the right half plane with slope other than ± 1 ; i.e. $x(t) = t$, $y(t) = at$ $t > 0$, $a \neq \pm 1$.

(b) Describe the level curves of $f(x, y)$.

(c) Sketch the graph of $f(x, y)$.