2S1 Problem Sheet 3

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QUESTION 1. Find the maxima and minima of the function $f(x, y) = x^2 + y^2$ under the constraint $3x^2 + 4xy + 12y^2 = 64$.

QUESTION 2. State the first and second partial derivative test for the local maxima, local minima and saddle points of a function $f(x, y)$ of two independent variables x and y and define a critical point.
Find the local minima and maxima and saddle points of the function

$$f(x, y) = \cos x \, \sin y.$$

QUESTION 3. A zeppelin[1] in the shape of an ellipsoid

$$6x^2 + 2y^2 + z^2 = 21$$

is flying over a bush fire. The temperature on its surface is found to be

$$T(x, y, z) = 60 - x^2y^2z^3.$$

Find the hottest points on the zeppelin's surface using the method of Lagrange multipliers.

QUESTION 4. Let f be a function of the two independent variables x and y with continuous second partial derivatives. Show that if $x = u+v$ and $y = u-v$, then

$$\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial u \partial v} + \frac{\partial^2 f}{\partial v^2} = a\frac{\partial^2 f}{\partial x^2} + b\frac{\partial^2 f}{\partial y^2}$$

for some constants a and b and determine a and b.

QUESTION 5. Let $f(x, y) = e^{x^2 - y}$.

(a) Find the directional derivative of f at the point P in the direction of the vector u in the following cases.

(i) $P = (1, 0)$, $u = (1, 2)$	(ii) $P = (2, 3)$, $u = (1, 4)$
(iii) $P = (3, 8)$, $u = (1, 6)$	(iv) $P = (-1, 0)$, $u = (1, -2)$

(b) Find a reason why you always obtained the same answer in part (a).

(c) Give a parametric description, i.e. $x(t) = \ldots$ and $y(t) = \ldots$, of the level curve of f on which $f(x, y) = e$.

(d) Try to use part (c) in order to answer part (b).

QUESTION 6. Let S be the unit sphere in $\mathbb{R}^3$, that is

$$S = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}.$$

Let B be a rectangular shaped box that fits inside S. What is the largest possible volume B can have? What is the shape of B that attains this maximal volume?

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[1] A zeppelin is a huge, helium filled, cigarillo shaped aircraft. They were popular until one exploded.