2S1 Problem Sheet 4

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QUESTION 1. Describe the constant u- and v-curves of the parametric surface defined by $r(u,v) = (\sin v, u, u + \cos v)$. Then compute the area of the surface when the parameter v is restricted by $0 \le v \le \frac{\pi}{8}$ and for each such v, $0 \le u \le \sin(2v)$.

QUESTION 2. Let R be the region bounded by the x-axis, the lines $x = 1$ and $x = 4$ and the graph of the function $g(x) = \sqrt{x}$. Determine the mass and centre of gravity of a flat object that occupies R and has density $\rho(x,y) = xy$.

QUESTION 3. Let n be a positive integer. Let $\mathcal{O}_n$ be a flat object occupying the region in the first quadrant of the xy-plane which is enclosed by the graphs of the functions $f(x) = x^n$ and $g(x) = \sqrt{x}$. Suppose the density of the object $\mathcal{O}_n$ is given by $\rho(x,y) = 9\,\frac{\sqrt{y}}{x}$. Let m_n denote the mass of the object $\mathcal{O}_n$ and determine $\lim_{n\to\infty} m_n$.

QUESTION 4. Evaluate the following double integrals.

$$\int_0^2 \int_{x^2}^4 x e^{y^2}\, dy\, dx \qquad \text{and} \qquad \int_0^1 \int_0^{\arccos y} \sqrt{1 + \sin x}\, dx\, dy$$

QUESTION 5. (a) Describe the surface $\mathcal{C}$ in 3-dimensional space which is determined by the equation $2x^2 + y^2 = 1$.

(b) Let $\mathcal{S}$ be the surface of the function $f(x,y) = \sqrt{1 - 2x^2}$ and find the volume of the solid which is bounded by the xy-plane and the surfaces $\mathcal{C}$ and $\mathcal{S}$.

(c) Compute the surface area of that part of $\mathcal{S}$ which is on the inside of the elliptical cylinder $\mathcal{C}$. *Hint:* You need to look up an integral in the log tables.

QUESTION 6. Sketch the curve $r = \sin(3\theta)$, where r and θ are polar coordinates, and determine the area enclosed by that curve.

QUESTION 7. A sheet of paper is $0.05722mm$ thick. Suppose you have a very large large sheet of paper, so that you can fold it repeatedly to half its previous size. How many times do you have to fold your paper to obtain a block that matches the height of the Dublin Spire which, I believe is $120m$ high? If your initial piece of paper is $1km^2$, what is the base size of the paper block that matches the Spire?

This figure *may* help to recover the construction of a pentagon.