2S1 Problem Sheet 6
April 26, 2005
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QUESTION 1. In the following cases give a complete description of the sample space and hence determine the underlying probability function.

(a) A coin and a die are tossed respectively rolled simultaneously.

(b) Two balls are drawn simultaneously from a bag which contains 4 green, 3 yellow, 2 blue and 1 red ball.

Now assign the following values to the balls: green=0, yellow=1, blue=2 and red=4. If one draw costs 1 Euro and the win is the product of the numbers on the two drawn balls in Euros, what is the expected win/loss of the game?

QUESTION 2. In an experiment four dice are rolled. Determine the probability that

(a) at most 2 dice show a six.

(b) at least 2 dice show a number less than or equal to 4.

(c) at least 2 but at most 3 dice show five or six.

(d) all but one die show the same number.

QUESTION 3. Let $\ln x$ denote the natural logarithm of x and define

$$f(x) = \begin{cases} ae^{-x}, & \text{if } 0 \le x \le \ln 2 \\ 0, & \text{otherwise} \end{cases}.$$

(a) Determine the mean and variance of a continuous random variable with density $f(x)$. Hint: integration by parts may help.

(b) Find the density of the random variable $X = X_1 + X_2$ where X_1 and X_2 both have density $f(x)$ and check that $P(1 \le X \le 2\ln 2) \approx 0.0969$.

QUESTION 4. Assume that the number of days between flowering and the resulting ripe cherry is normally distributed.

(a) A sample of size 50 had a mean of 96 days and standard deviation 14 days. Find a confidence interval for the mean and the variance of the underlying distribution at the 98% level.

(b) Under the assumption that the underlying distribution has variance 121 days, what sample size would guarantee a confidence interval at the 97.5% level for the mean of length at most 6 days?

QUESTION 5. Suppose that on average there are 15 hours of rain during the first week of May in Ireland. What is the probability that this year, during the first week of May, there will be (a) no rain, (b) more than 6 hours of rain, (c) between 10 and 12 hours of rain, (d) less than 4 hours of rain?