CS3304 Logic – Problem Sheet 2

September 20, 2016, Lecturer: Claas Röver

QUESTION 1. James, Kevin and Michael are suspected of income tax evasion. They testify under oath as follows.

JAMES: Kevin is guilty and Michael is innocent.

KEVIN: If James is guilty, then so is Michael.

MICHAEL: I'm innocent, but at least one of the others is guilty.

Let J, K, M be the statements, "James is innocent", "Kevin is innocent", "Michael is innocent", respectively. Express the testimony of each suspect by a proposition, and write out truth tables for these three statements in parallel columns. Now answer the following questions.

(a) Are the testimonies of the three suspects consistent?

(b) The testimony of one suspect follows from that of another. Which from which?

(c) Assuming everybody is innocent, who committed perjury?

(d) Assuming everyone's testimony is true, who is innocent and who is guilty?

(e) Assuming that the innocent told the truth and the guilty told lies, who is innocent and who is guilty?

QUESTION 2. Decide whether the set of propositions

$$A \to C,\ C \vee B \vee D,\ (D \vee C) \to (B \to C) \text{ and } \neg C \vee \neg A$$

is (a) inconsistent, (b) satisfyable, (c) a tautology.

QUESTION 3. Using semantic tableaux, decide whether the following arguments are valid.

(a) If Aoife is 18 years old and holds a drivers licence, then she is allowed to drive a car. Aoife is not allowed to drive a car. Therefore she is not 18 years old.

(b) If Aoife is 18 years old, then if she holds a drivers licence, then she is allowed to drive a car. Aoife is not allowed to drive a car. Therefore she does not hold a drivers licence.

(c) If Aoife is 18 years old, then if she holds a drivers licence, then she is allowed to drive a car. Aiofe is 18 years old. Aiofe is not allowed to drive a car. Therefore she does not hold a drivers licence.

QUESTION 4. True or false? In the following A is a statement and $\mathcal{S}$ is a set of statements.

(a) Every tautology is satisfyable.

(b) If $\mathcal{S}$ is inconsistent, then $\mathcal{S}$ is satisfyable.

(c) A is a tautology if and only if $\neg A$ is unsatisfyable.

(d) If $\mathcal{S}$ is inconsistent, then $\mathcal{S} \cup \{A\}$ is inconsistent.

(e) If $\mathcal{S}$ is satisfyable, then $\mathcal{S} \cup \{A\}$ is satisfyable.

(f) If $\mathcal{S} \cup \{A\}$ is satisfyable, then $\mathcal{S}$ is satisfyable.

(g) If $\mathcal{S} \cup \{A\}$ is inconsistent, then $\mathcal{S}$ is inconsistent.