CS3304 Logic – Problem Sheet 5

October 14, 2016, Lecturer: Claas Röver

QUESTION 1. State the three axioms, or rather axiom schema, of axiomatic propositional logic (APL for short). Then decide which of the following **wff** are instances of these axioms.

(a) $((\neg p_1) \to ((p_1 \to p_2) \to (\neg p_1)))$

(b) $((((\neg q) \to (\neg(\neg p))) \to (\neg((p \to q) \to r))) \to (A \to (q \to (\neg p))))$

(c) $((((p \to q) \to r) \to ((\neg(\neg p)) \to (\neg q))) \to ((((p \to q) \to r) \to (\neg(\neg p))) \to (((p \to q) \to r) \to (\neg q))))$

(d) $(((\neg(\neg p_2)) \to (\neg((q_2 \to p_1) \to q_1))) \to (((q_2 \to p_1) \to q_1) \to (\neg p_2)))$

(e) $(((\neg(p_0 \to p_1)) \to (\neg(p_2 \to (p_0 \to p_1)))) \to ((p_2 \to (p_0 \to p_1)) \to (p_0 \to p_1)))$

(f) $((\neg(p_0 \to p_1) \to (p_2 \to (p_0 \to p_1))) \to ((\neg(p_2 \to (p_0 \to p_1))) \to (p_0 \to p_1)))$

QUESTION 2. Find a deduction in APL of the following useful theorem, which is known as *transitivity of implication* (TI for short). *Hint:* Use the Deduction Theorem (DT for short).

$$\{(A \to B),\, (B \to C)\} \vdash (A \to C)$$

QUESTION 3. Here is a deduction of $\vdash \neg\neg A \to A$ using TI.

1.	$\neg\neg A \to (\neg\neg\neg\neg A \to \neg\neg A)$	Axiom 1
2.	$(\neg\neg\neg\neg A \to \neg\neg A) \to (\neg A \to \neg\neg\neg A)$	Axiom 3
3.	$\neg\neg A \to (\neg A \to \neg\neg\neg A)$	TI
4.	$(\neg A \to \neg\neg\neg A) \to (\neg\neg A \to A)$	Axiom 3
5.	$\neg\neg A \to (\neg\neg A \to A)$	TI
6.	$(\neg\neg A \to (\neg\neg A \to A)) \to ((\neg\neg A \to \neg\neg A) \to (\neg\neg A \to A))$	Axiom 2
7.	$(\neg\neg A \to \neg\neg A) \to (\neg\neg A \to A)$	MP 5 & 6
8.	$\neg\neg A \to \neg\neg A$	Instance of $A \to A$ (proved in lecture)
9.	$\neg\neg A \to A$	MP 7 & 8

(a) Modify the above deduction sequence from below line 5, replacing the use of Axiom 2 by an invocation of the Deduction Theorem.

(b) Find a deduction of $\vdash \neg\neg A \to A$ using an instance of the theorem $\vdash \neg A \to (A \to B)$, which was proved in the lectures, and at most one (instance of an) axiom.

QUESTION 4. Find deductions of the following in APL. You may use TI, DT and previous results of this question.

(a) $\{B,\, A \to (B \to C)\} \vdash A \to C$

(b) $\vdash (A \to (B \to C)) \to (B \to (A \to C))$

(c) $\vdash (A \to B) \to (\neg B \to \neg A)$

(d) $\vdash A \to \neg\neg A$

(e) $\vdash \neg(A \to B) \to \neg B$