CS3304 Logic – Assignment 2

October 21, 2016, Lecturer: Claas Röver

Hand in your solution at the beginning of the lecture on **Friday, 4 Nov 2016**.

QUESTION 1. Give deductions of the following, using the sequent rules. The rules are available for download on the course web page.

(a) $\Rightarrow \neg(\neg A \wedge B) \to A \vee \neg B$

(b) $\Rightarrow (A \to (B \to C)) \to ((A \to B) \to (A \to C))$

(c) $A \to (B \to C), A \vee B \Rightarrow A \to C, B \to C$

QUESTION 2. (a) State the definition of a valuation $v\colon \mathcal{W} \to \{T, F\}$, where $\mathcal{W}$ denotes the set of well formed formulæ.

(b) Verify that a valuation assigns the same truth values to $A \vee B$ and $A \wedge B$ as the truth table definitions, when they are defined as follows, in terms of $\to$ and $\neg$.

$A \wedge B$ if and only if $\neg(A \to \neg B)$

$A \vee B$ if and only if $\neg A \to B$

(c) Prove that $\neg p \to (p \to (q \wedge \neg q))$ is a tautology by showing that its value is T under every valuation.

QUESTION 3. (a) State the three axioms, or rather axiom schema, of axiomatic propositional logic (APL for short).

(b) State and prove the Deduction Theroem.

(c) Use the Deduction Theorem (and the axioms) to prove

$$\vdash A \to ((A \to B) \to ((A \to (B \to C)) \to C)).$$

QUESTION 4. Prove the folllowing propositions in APL, using only the axioms, transitivity of implication, the Deduction Theorem and the fact $\vdash \neg A \to (A \to B)$.

(a) $\vdash A \to (B \to (\neg B \to A))$

(b) $\{\neg A,\, A \to (\neg B \to C)\} \vdash A \to C$

(c) $\vdash \neg(A \to B) \to (B \to A)$