CS3304 Logic – Problem Sheet 7

November 4, 2016, Lecturer: Claas Röver

QUESTION 1. (a) Convert the following formulæ into conjunctive normal form.

(i) $\neg(p \rightarrow (q \vee r))$ (ii) $(\neg p) \vee (q \leftrightarrow \neg r)$ (iii) $(p \rightarrow q) \rightarrow ((\neg p \wedge r) \vee \neg q)$

(iv) $p \vee (r \rightarrow \neg s)$ (v) $(s \wedge q) \vee \neg(s \wedge r)$ (vi) $(q \vee \neg p) \rightarrow (s \rightarrow (\neg q \wedge p))$

(b) Use resolution to decide whether the two sets of formulæ (i)–(iii) and (iv)–(vi) from part (a) are satisfyable. What about the set of formulæ (i)–(vi)?

QUESTION 2. Prove the following theorems using resolution.

(a) $\vdash (p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$

(b) $\vdash ((p \rightarrow (q \rightarrow r)) \wedge p) \rightarrow \neg(q \wedge \neg r)$

QUESTION 3. Prove the **Unit Clause Rule**: Let S be a set of clauses. Suppose S contains a unit clause $\{\lambda\}$, for some literal λ, and define

$$S' = \{C \setminus \{\neg\lambda\} \mid C \in S,\ \lambda \notin C\};$$

that is, S' is obtained from S by deleting all clauses containing λ and removing $\neg\lambda$ from the remaining clauses. Then S is satisfyable if and only if S' is satisfyable.

QUESTION 4. Prove the **Pure Literal Rule**: Let S be a set of clauses. Suppose that a literal λ is contained in some clause in S but $\neg\lambda$ is not contained in any clause in S, and define

$$S' = \{C \mid C \in S,\ \lambda \notin C\};$$

that is S' obtained from S by deleting all clauses containing λ. Then S is satisfyable if and only if S' is satisfyable.

QUESTION 5. Use resolution to decide whether the following is a valid argument. If $A \rightarrow B$, $\neg A \vee C \vee D$, $\neg C \vee (D \wedge A)$ and $(C \wedge \neg D) \rightarrow \neg E$ hold, then $\neg D \rightarrow B$ also holds.

QUESTION 6. Use resolution to prove that a triangle is a 3-colourable but not 2-colourable graph.

QUESTION 7. Assume that R is a unary and P a binary relation, c is a constant and f is a binary function. As usual, x and y are variables. Decide which of the following are well formed fromulæ of predicate logic,

(a) $(P(x, y) \wedge \neg R(x))$ (b) $\exists x R(x) f(x, y)$ (c) $\forall y(R(c, y) \rightarrow Px)$

(d) $\forall x \exists x P(f(x, c), x)$ (e) $\exists x(P(x) \rightarrow \forall y R(y))$ (f) $\forall x f(R(y), x)$