# Sequent Rules

Below capital Greek letters denote *finite* sequences of statements, while capital Roman letters stand for statements. A sequent is of the form $\Gamma \Rightarrow \Delta$ and purely formal. However, it helps to interpret it as:

> ***If** a choice of truth values makes all statements in $\Gamma$ true, **then** it makes at least one of the statements in $\Delta$ true.*

or equivalently:

> *For every choice of truth values at least one of statements in $\Gamma$ is false* **or** *at least one of the statements in $\Delta$ is true.*

$$\textbf{R}\wedge: \ \frac{\Gamma_1 \Rightarrow A, \Delta_1 \qquad \Gamma_2 \Rightarrow B, \Delta_2}{\Gamma_1, \Gamma_2 \Rightarrow A \wedge B, \Delta_1, \Delta_2} \qquad \textbf{L}\wedge: \ \frac{\Gamma, A \Rightarrow \Delta}{\Gamma, A \wedge B \Rightarrow \Delta} \qquad \frac{\Gamma, B \Rightarrow \Delta}{\Gamma, A \wedge B \Rightarrow \Delta}$$

$$\textbf{L}\vee: \ \frac{\Gamma_1, A \Rightarrow \Delta_1 \qquad \Gamma_2, B \Rightarrow \Delta_2}{\Gamma_1, \Gamma_2, A \vee B \Rightarrow \Delta_1, \Delta_2} \qquad \textbf{R}\vee: \ \frac{\Gamma \Rightarrow A, \Delta}{\Gamma \Rightarrow A \vee B, \Delta} \qquad \frac{\Gamma \Rightarrow B, \Delta}{\Gamma \Rightarrow A \vee B, \Delta}$$

$$\textbf{L}\to: \ \frac{\Gamma_1 \Rightarrow A, \Delta_1 \qquad \Gamma_2, B \Rightarrow \Delta_2}{\Gamma_1, \Gamma_2, A \to B \Rightarrow \Delta_1, \Delta_2} \qquad \textbf{R}\to: \ \frac{\Gamma, A \Rightarrow B, \Delta}{\Gamma \Rightarrow A \to B, \Delta}$$

$$\textbf{L}\neg: \ \frac{\Gamma \Rightarrow A, \Delta}{\Gamma, \neg A \Rightarrow \Delta} \qquad \textbf{R}\neg: \ \frac{\Gamma, A \Rightarrow \Delta}{\Gamma \Rightarrow \neg A, \Delta} \qquad \textbf{LT}: \ \frac{\Gamma \Rightarrow \Delta}{\Gamma, A \Rightarrow \Delta} \qquad \textbf{RT}: \ \frac{\Gamma \Rightarrow \Delta}{\Gamma \Rightarrow A, \Delta}$$

$$\textbf{LC}: \ \frac{\Gamma, A, A \Rightarrow \Delta}{\Gamma, A \Rightarrow \Delta} \qquad \textbf{RC}: \ \frac{\Gamma \Rightarrow A, A, \Delta}{\Gamma \Rightarrow A, \Delta} \qquad \textbf{LR}: \ \frac{\Gamma, A, B \Rightarrow \Delta}{\Gamma, B, A \Rightarrow \Delta} \qquad \textbf{RR}: \ \frac{\Gamma \Rightarrow A, B, \Delta}{\Gamma \Rightarrow B, A, \Delta}$$

$$\textbf{Cut}: \ \frac{\Gamma_1 \Rightarrow A, \Delta_1 \qquad \Gamma_2, A \Rightarrow \Delta_2}{\Gamma_1, \Gamma_2 \Rightarrow \Delta_1, \Delta_2} \qquad \textbf{Id}: \ A \Rightarrow A$$

Here is an example, how to prove $\Rightarrow \neg(\neg A \vee B) \to (A \vee B)$, using the sequent rules.

$$\begin{array}{rl}
A \Rightarrow A & \textbf{Id} \\
\hline
\Rightarrow \neg A, A & \textbf{R}\neg \\
\hline
\Rightarrow \neg A \vee B, A & \textbf{R}\vee \\
\hline
\neg(\neg A \vee B) \Rightarrow A & \textbf{L}\neg \\
\hline
\neg(\neg A \vee B) \Rightarrow A \vee B & \textbf{R}\vee \\
\hline
\Rightarrow \neg(\neg A \vee B) \to (A \vee B) & \textbf{R}\to
\end{array}$$

**HEURISTIC:** Work from the conclusion and see how a non-atomic statement could be falsified. Put the constituent that needs to be true on the left and the constituent that needs to be false on the right side of the $\Rightarrow$.