MA203 Linear Algebra – Problem Sheet 3

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QUESTION 1. Given the matrices

$$A = \begin{pmatrix} 2 & 8 & -4 \\ -1 & 3 & -2 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 & 0 \\ -2 & -4 & 5 \\ -3 & 0 & 2 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} 5 & 6 \\ 4 & 5 \\ 3 & 4 \\ 2 & 3 \end{pmatrix},$$

decide for each of the products AB, BA, AC, CA, BC, CB, ABC, BCA and CAB whether it is defined, and if so, then calculate the product and say what its dimensions are.

QUESTION 2. Let $A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$, $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and $w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ be an arbitrary 3×3 matrix and two arbitrary 3-dimensional column vectors. Verify by direct calculation that $A(v + w) = Av + Aw$.

QUESTION 3. Let A be the matrix from Question 2. Calculate $A \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} A$.

Can you guess what the results of $A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $A \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} A$ and $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} A$ will be?

Check whether you were right.

QUESTION 4. Find all solutions of the matrix equation

$$\begin{pmatrix} -1 & 2 & -4 & 0 \\ 3 & -2 & 1 & 3 \\ -2 & 0 & 2 & 3 \\ 1 & 3 & 0 & -1 \end{pmatrix} \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -8 \\ 18 \\ 7 \\ -4 \end{pmatrix}.$$

QUESTION 5. Calculate $\begin{pmatrix} -5 & 40 & -30 & 30 \\ 11 & -2 & 23 & 63 \\ -47 & -11 & 19 & 24 \\ 28 & 34 & 39 & 4 \end{pmatrix} \begin{pmatrix} -1 & 2 & -4 & 0 \\ 3 & -2 & 1 & 3 \\ -2 & 0 & 2 & 3 \\ 1 & 3 & 0 & -1 \end{pmatrix}$. Use this result to verify your answer to Question 4.