MA211 Calculus I – Problem Sheet 3

QUESTION 1. For each of the following statements decide whether it is true or false.

(a) The function $f(x) = x^3 - 2x^2 - 4$ has an inverse function.

(b) Every one-to-one function $f : \mathbb{R} \to \mathbb{R}$ has range equal to $\mathbb{R}$.

(c) The hyperbolic sine is a periodic function, just like the trigonometric sine.

(d) The hyperbolic sine is an odd and one-to-one function.

QUESTION 2. Let $f(x) = Ae^{kx} + Be^{-kx}$ with $A, B, k \in \mathbb{R}$ and $k > 0$.

(a) Verify that $f''(x) = k^2 f(x)$.

(b) Find constants C and D such that $f(x) = C \cosh(x) + D \sinh(x)$ holds for all $x \in \mathbb{R}$.

(c) Determine A and B so that $f(0) = 6$ and $f'(0) = 0$ holds.

(d) Give a condition ensuring that $f(x) = 0$ has a solution.

QUESTION 3. Use integration by parts to derive the following formulae.

$$(a) \quad \int \sin^n(x)\, dx = -\frac{1}{n} \cos(x) \sin^{n-1}(x) + \frac{n-1}{n} \int \sin^{n-2}(x)\, dx$$

$$(b) \quad \int \sin^{-1}(x)\, dx = x \sin^{-1}(x) + \sqrt{1 - x^2} + C, \quad C = const.$$

QUESTION 4. Find the following indefinite integrals.

(a) $\int x^2 \ln(x)\, dx$

(b) $\int (2x - 1)e^{x^2 - x + 1}\, dx$

(c) $\int \sin^5(x) \cos(x)\, dx$

(d) $\int e^x \cos(x)\, dx$

QUESTION 5. Evaluate the following integrals.

(a) $\int_0^1 \frac{3}{\sqrt{x^2 + 4x + 3}}\, dx$

(b) $\int_{-1}^1 \cosh(x)\, dx$

(c) $\int_0^{\ln(3)} x \sinh(x^2 + \ln(2))\, dx$