MA211 Calculus I – Problem Sheet 4

October 3, 2016, Lecturer: Claas Röver

QUESTION 1. Assume that the sequences $(a_i)_{i\in\mathbb{N}}$ and $(b_i)_{i\in\mathbb{N}}$ are both convergent and prove the following.

(a) The sequence $(s_i)_{i\in\mathbb{N}}$ defined $s_i = a_i + b_i$ is convergent and

$$\lim_{i\to\infty} s_i = (\lim_{i\to\infty} a_i) + (\lim_{i\to\infty} b_i).$$

In other words, the sum of two convergent sequences is convergent with limit equal to the sum of the limits.

(b) The sequence $(p_i)_{i\in\mathbb{N}}$ defined $p_i = a_i b_i$ is convergent and

$$\lim_{i\to\infty} p_i = (\lim_{i\to\infty} a_i)(\lim_{i\to\infty} b_i).$$

In other words, the product of two convergent sequences is convergent with limit equal to the product of the limits.

QUESTION 2. Find examples for each of the following situations.

(a) Two divergent sequences (a_i) and (b_i) such that the sequence $(a_i b_i)$ is convergent.

(b) A convergent sequence (a_i) and a divergent sequence (b_i) such that the sequence $(a_i b_i)$ is (i) convergent or (ii) divergent.

(c) Two convergent sequences (a_i) and (b_i) with $b_i \neq 0$ for $i \in \mathbb{N}$ such that the sequence (a_i/b_i) is divergent.

QUESTION 3. For each of the following improper integrals, decide whether it exists and if so calculate its value.

$$\text{(a)} \int_0^2 \frac{1}{x^2}\,dx \qquad \text{(b)} \int_0^4 \frac{1}{\sqrt{x}}\,dx \qquad \text{(c)} \int_0^\infty \sin(x)\,dx \qquad \text{(d)} \int_{-\infty}^1 e^{x-1}\,dx$$

$$\text{(e)} \int_0^\infty \frac{1}{\sqrt{x^3+x}}\,dx \qquad \text{(f)} \int_0^1 x\ln(x)\,dx \qquad \text{(g)} \int_0^\infty xe^{-x}\,dx \qquad \text{(h)} \int_{-\infty}^\infty xe^{-x^2}\,dx$$