MA211 Calculus I – Assignment 2

October 10, 2016, Lecturer: Claas Röver

Hand in your solution at the beginning of the lecture on **Monday, 17 Oct 2016**.

QUESTION 1. Let f^{-1} be the inverse function of a one-to-one function f.

(a) State the two defining equations for f^{-1}.

(b) Using part (a) and the chain rule, show that

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}.$$

(c) Use part (b) to show that

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}} \quad \text{for } -1 \leq x \leq 1$$

QUESTION 2. Let $(a_i)_{i\in\mathbb{N}}$ be a sequence.

(a) State the definition for the convergence of $(a_i)_{i\in\mathbb{N}}$ with limit L.

(b) Assume that $\lim_{n\to\infty} a_n = L$ for some finite L. Prove from first principles that the sequence of consecutive differences $d_i = a_i - a_{i-1}$ converges with limit zero.
Hint: Use the triangle inequality.

QUESTION 3. Evaluate the following improper integrals.

$$\text{(a)} \int_2^\infty \frac{1}{x^3}\,dx \qquad \text{(b)} \int_1^2 \frac{1}{\sqrt{2-x}}\,dx \qquad \text{(c)} \int_0^1 x\ln(x)\,dx$$

QUESTION 4. Show that the following improper integrals diverge.

$$\text{(a)} \int_0^\infty \frac{1}{4+x^2}\,dx \qquad \text{(b)} \int_2^\infty \frac{1}{\ln(x)}\,dx$$

QUESTION 5. Show that

$$\int_{-1}^{1} \sqrt{\frac{1+x}{1-x}}\,dx = \pi.$$

Hint: Have a look at Question 1 (c).