MA211 Calculus I – Problem Sheet 7

October 24, 2016, Lecturer: Claas Röver

QUESTION 1. A ball, when dropped vertically, bounces back up to 60% of the height it was dropped from. How much distance does the ball travel if it is allowed to bounce indefinitely after dropping it from $3\,\mathrm{m}$ height?

QUESTION 2. Prove the following statements. *Hint:* Use facts about limits of sums and differences of converging sequences from Problem Sheet 4.

(a) The series $\sum_{n=1}^{\infty} a_n$ converges if and only if, for every $N \geq 1$, the series $\sum_{n=N}^{\infty} a_n$ converges

(b) If the series $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

QUESTION 3. You may use the fact that a bounded nondecreasing sequence is convergent, i.e. if there exists K such that $a_n \leq a_{n+1} \leq K$ for all $n \geq 1$, then $\lim_{n\to\infty} a_n$ exits.

(a) A series $\sum_{n=1}^{\infty} a_n$, with all $a_n \geq 1$, converges, if its sequence of partial sums is bounded.

(b) **Estimation Theorem:** Let $\{a_n\}$ and $\{b_n\}$ be sequences. If there exists a constant K with $a_n \leq K b_n$, then

(i) if $\sum_{n=1}^{\infty} b_n$ converges, then so does $\sum_{n=1}^{\infty} a_n$; and

(ii) if $\sum_{n=1}^{\infty} a_n$ diverges, then so does $\sum_{n=1}^{\infty} b_n$.

QUESTION 4. Determine the sum of the following telescoping series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$

QUESTION 5. For each of the following series, decide whether it converges or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2}{4n^2+3n+1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{7}{n^2+\sqrt{n}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{(\ln(n))^3}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^4}{n!}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^n-n^3}$ (f) $\displaystyle\sum_{n=1}^{\infty} e^{-n} n!$