MA343 Group Theory I – Problem Sheet 4

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QUESTION 1. Show that a group G whose order is a prime number must be cyclic.

QUESTION 2. Find the isomorphism type of the group generated by the matrices

$$a = \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix} \qquad \text{and} \qquad b = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

QUESTION 3. Decide which of the following maps are homomorphisms of groups, and for those that are determine the kernel.

(a) $\det\colon GL(n, \mathbb{R}) \longrightarrow \mathbb{R},\ M \mapsto \det(M)$, where $GL(n, \mathbb{R})$ is the group of $n \times n$ matrices with real entries and non-zero determinant, and $\det(M)$ is the determinant of M.

(b) ${}^{-1}\colon G \longrightarrow G,\ g \mapsto g^{-1}$, where $G = \mathrm{Sym}(\mathrm{X})$ and $|X| > 2$.

(c) $\exp\colon (\mathbb{R}, +) \longrightarrow (\mathbb{C}, \cdot),\ x \mapsto e^{ix/(2\pi)}$

QUESTION 4. Find all subgroups and quotients of D_6, the dihedral group of order twelve.

QUESTION 5. Let G be a group and A an abelian group. Show that for every homomorphism $\alpha\colon G \to A$, we have $g^{-1}h^{-1}gh \in \ker(\alpha)$ for all $g, h \in G$. Hence, or otherwise, show that if D is a dihedral group, then the largest abelian quotient of S has order two.

QUESTION 6. Determine the automorphism group of a cyclic group of order ten. What is its order? Is it cyclic?