

MA343 Group Theory I – Assignment 3

November 4, 2011, Lecturer: Claas Röver

QUESTION 1. Suppose that $\alpha: G \rightarrow H$ is a surjective homomorphism of groups. Let U be a subgroup of H and prove the following.

- (a) The pre-image of U is a subgroup of G containing $\ker(\alpha)$.
- (b) If U is normal in H , then the pre-image of U is normal in G .
- (c) The image of the centre of G under α is contained in the centre of H , i.e. $Z(G)^\alpha \subset Z(H)$.

QUESTION 2. (a) Find an example of a group G with subgroups K and N such that N is normal in G , K is normal in N but K is not normal in G .

- (b) Find groups G and H and a surjective homomorphism $\alpha: G \rightarrow H$ such that $Z(G)^\alpha \neq Z(H)$.

QUESTION 3. (a) Define the Cayley graph of a group G with respect to a generating set $S \subset G$.

- (b) Draw the Cayley graph of $D_5 = \langle \sigma, \tau \mid \sigma^2, \tau^2, (\sigma\tau)^5 \rangle$ with respect to each of the generating sets $S = \{\sigma, \tau\}$ and $T = \{\sigma, \rho\}$, where $\rho = \sigma\tau$.
- (c) Draw the Cayley graph of the quaternion group $\langle i, j, k \mid i^2 = j^2 = k^2 = ijk, i^4 = 1 \rangle$ with respect to $S = \{i, j, k\}$.

QUESTION 4. Determine all normal subgroups and all quotients of S_4 , the symmetric group of degree 4.

QUESTION 5. Determine the automorphism group of S_3 , the symmetric group of degree three.

Hand in your solutions at the beginning of the lecture on Thursday, November 17, 2011.