MA500 Advanced Group Theory – Assignment 1

February 20, 2017, Lecturer: Claas Röver

Hand in your solution at the beginning of the lecture on **Wednesday, 1 March 20167**.

TASK 1. Write up your solution to Task 4 on Problem Sheet 1.

TASK 2. Write up your solution to Task 5 on Problem Sheet 1.

TASK 3. Give an example of a group G and surjective homomorphism $\alpha\colon G \longrightarrow G$ such that α is not injective. *Hint:* Have a look at Problem Sheet 2.

TASK 4. The Baumslag–Solitar groups are one-relator groups defined by

$$\mathrm{BS}(n,m) = \langle a, t \mid t^{-1}a^n t = a^m \rangle,$$

where n and m are non-zero integers.

(a) Show that the map ψ define by

$$a^\psi = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \text{ and } t^\psi = \begin{pmatrix} \frac{n}{m} & 0 \\ 0 & 1 \end{pmatrix}$$

extends to a homomorphism from $\mathrm{BS}(n,m)$ to $\mathrm{GL}_2(\mathbb{Q})$.

(b) Prove that the quotient of $\mathrm{BS}(n,m)$ by its commutator subgroup is (isomorphic to) the direct product of an infinite cyclic group with a finite cyclic group of order $|m-n|$.

TASK 5. Finding finitely generated examples with the property demanded in Task 3 is much harder.

(a) Show that the map φ with $a^\varphi = a^n$ and $t^\varphi = t$ from $\mathrm{BS}(2,3)$ to itself extends to a surjective homomorphism.

(b) Now show that $\mathrm{BS}(2,3)$ has the property asked for in Task 3, by firstly verifying that $w = a^{-1}t^{-1}atat^{-1}a^{-1}t \,(= [a,t][a,t]^{-a^{-1}})$ is in the kernel of φ and secondly applying Britton's lemma to show that $w \neq 1$.

(c) Verify that the element w defined in part (b) is in the kernel of the homomorphism ψ form Task 4.