Introduction to MA101 semester 2

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MA100

Welcome to semester 2

lecturer:	Emil Sköldberg, `emil.skoldberg@nuigalway.ie` Room C212 Áras de Brún
lectures:	Monday 1–2 in Kirwan Tuesday 10–11 in Kirwan
tutorials:	Times and venues to be arranged Start next week.
textbook:	Stewart: "*Calculus, Early Transcendentals*"
website:	`http://www.nuigalway.ie/~emil/teaching/ma100/`

Assessment

The mark for MA101 and the calculus component of MA160 is computed as follows:

Semester 1 homework:	8%
Semester 2 homework:	12%
Christmas test:	10%
Summer exam:	70%

The homework assignments during semester 2 will use the WebWork system, just as for semester 1.

Topics

There are two main topics of this semester:

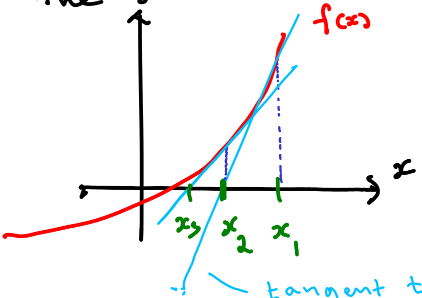
- *Integration* This will take up most of the semester. We will treat the construction of the Riemann integral, the fundamental theorem of calculus, techniques for integration and applications of integration.
- *Differential Equations* We will only touch upon this vast subject, and look at differential equations that model population growth.

Newton's method

Suppose we want to solve an equation

$$f(x) = 0$$

and suppose we have an initial guess x_1 for the solution. How can we now get a better estimate of the solution?



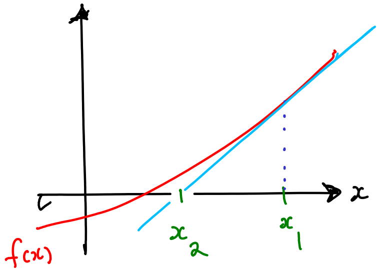
tangent to $f(x)$ at $(x_1, f(x_1))$

- Draw tangent to $f(x)$ at $(x_1, f(x_1))$
- Find tangent's intercept with x-axis. Call it x_2

Repeat

Newton's method

Let us now find a formula for x_2.



The equation for the tangent line is now

$$y = f'(x_1)(x - x_1) + f(x_1)$$
$$= f'(x_1)\,x + \left(f(x_1) - f'(x_1)x_1\right)$$

Next, we solve

$$0 = f'(x_1)(x - x_1) + f(x_1)$$
$$f'(x_1)\,x = f'(x_1)\,x_1 - f(x_1)$$
$$x_2 := x = x_1 - \frac{f(x_1)}{f'(x_1)}.$$

x_2 is now our next approximation.

Newton's method

The general method:

To find approximate solution to

$f(x) = 0$

given a guess x_1:

Set, for $n = 1, 2, 3, \ldots$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Example

Using $x_1 = 2$, find the third approximation to the root of the equation $x^3 - 2x - 5 = 0$.

Now $f(x) = x^3 - 2x - 5$

so $f'(x) = 3x^2 - 2$

We can now run Newton's method:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{2^3 - 2\cdot 2 - 5}{3\cdot 2^2 - 2} = 2.1$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.1 - \frac{2.1^3 - 2\cdot 2.1 - 5}{3\cdot 2.1^2 - 2} \approx 2.0946$$

The third approx. is thus 2.0946

Example

Use Newton's method to find $\sqrt[6]{2}$ correct to eight decimal places.

$\sqrt[6]{2}$ is the solution to the equation

$$x^6 - 2 = 0$$

Let $f(x) = x^6 - 2$, so $f'(x) = 6x^5$

Newton's method for this problem becomes

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^6 - 2}{6x_n^5}$$

Starting with $x_1 = 1$ we get

$x_1 = 1$
$x_2 \approx 1.1666667$
$x_3 \approx 1.2644368$
$\vdots$
$x_5 \approx 1.12246205$
$x_6 \approx 1.12246205$