# Volumes

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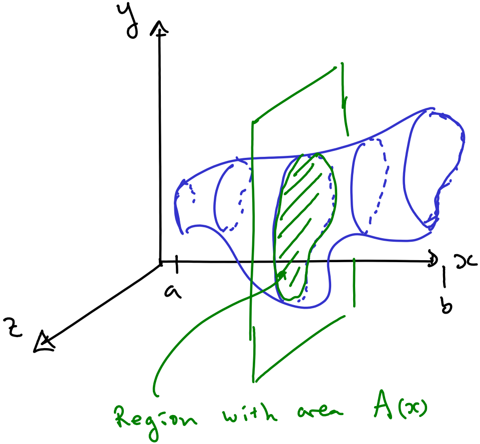
# Definition of volume

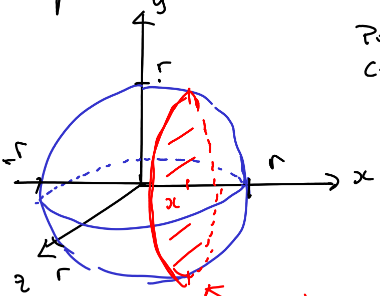



We will define the volume of the solid by

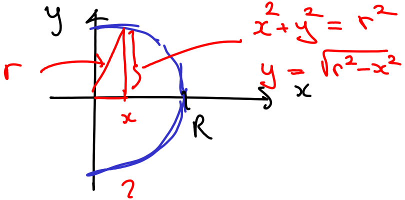
$$\boxed{\int_a^b A(x)\,dx}$$

# The volume of a sphere

Let us use the definition to calculate the volume of a sphere with radius $r$




Position the sphere so its centre is at the origin




What is this area?

It is a circle of radius $\sqrt{r^2-x^2}$

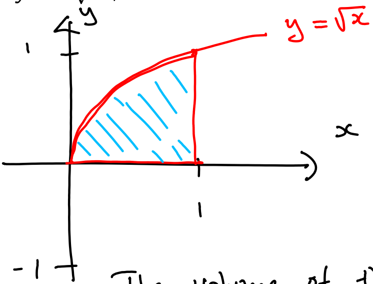
so its area is $\pi\cdot(r^2-x^2)$

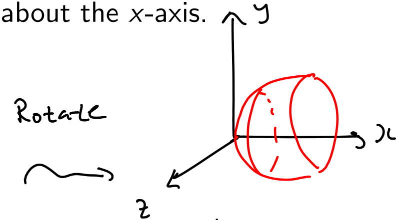
Thus, the volume of the sphere is

$$\int_{-r}^{r}\pi\cdot(r^2-x^2)\,dx=\pi\int_{-r}^{r}r^2-x^2\,dx=\pi\left[r^2x-\frac{x^3}{3}\right]_{-r}^{r}=\pi\left(r^3-\frac{r^3}{3}+r^3-\frac{r^3}{3}\right)=\frac{4\pi r^3}{3}$$

# Example

Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, the $x$-axis and $x = 1$ about the $x$-axis.



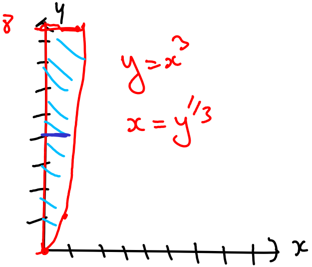

The volume of the region is $\int_0^1 A(x)\,dx$

The cuts are all circular of radius $\sqrt{x}$, so $A(x) = \pi \cdot x$

so the volume is $\int_0^1 \pi x\,dx = \left[\frac{\pi x^2}{2}\right]_0^1 = \frac{\pi \cdot 1}{2} - \frac{\pi \cdot 0}{2} = \underline{\frac{\pi}{2}}$

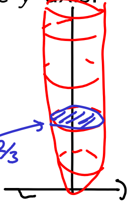
# Example

Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$ and $x = 0$ about the $y$-axis.




Rotate

$A(y) = \pi \cdot y^{2/3}$



To find the volume we integrate with respect to $y$!

The volume is $\int_0^8 A(y)\, dy =$

$$= \int_0^8 \pi y^{2/3}\, dy = \pi \left[ \frac{3 y^{5/3}}{5} \right]_0^8 = \pi \left( \frac{3 \cdot 8^{5/3}}{5} - \frac{3 \cdot 0^{5/3}}{5} \right) = \frac{\pi \cdot 3 \cdot 32}{5}$$

$$= \frac{96}{5} \pi$$