#### Strategies of integration

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#### MA100

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# A four-step plan

- Simplify the integrand.
- 2 Look for an obvious substitution.
- Olassify the integrand accoring to its form
  - Trigonometric functions.
  - 2 Rational functions.
  - Integration by parts.
- Try again!

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# Simplify the integrand

$$\int \sqrt{x}(1+\sqrt{x}) dx$$

$$\int \sqrt{x}(1+\sqrt{x}) dx = \int (\sqrt{x} + x) dx$$

$$= \int \sqrt{x^2}(1+\sqrt{x}) dx = \int (x^{1/2} + x) dx$$

$$= \frac{x^{3/2}}{3/2} + \frac{x^2}{2} + C = \frac{2}{3} + \frac{3}{2} + \frac{1}{2} + C$$

$$\int (\sin x + \cos x)^2 dx$$

$$\int (\sin x + \cos x)^2 dx = \int (\sin^2 x + 2\sin x \cos x + \cos^2 x) dx =$$

$$= \int 1 + 2\sin x \cos x dx = \int (1+\sin 2x) dx = x - \frac{1}{2} \cosh x + C$$

$$\int x \sin 2x dx = \int (1+\sin 2x) dx = x - \frac{1}{2} \cosh x + C$$

## Look for an obvious substitution

$$\int \frac{x^2}{x^3 - 1} dx$$

$$\int \frac{x}{x^3 - 1} dx = \begin{bmatrix} u = x^3 - 1 \\ du = 3x^2 \\ du = 3x^2 \end{bmatrix}$$

$$\int \frac{1}{n} \frac{1}{3} \frac{3^{2}}{3^{2}} \frac{dx}{dx} = \frac{1}{3} \int \frac{1}{n} \frac{dx}{dx} = \frac{1}{3} \ln |u| + C$$
$$= \frac{1}{3} \ln |x^{2} - 1| + C$$

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## Classify the integrand according to its form

Rational functions:

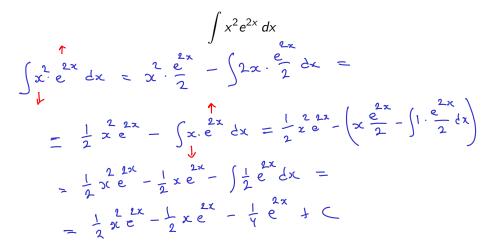
$$\int \frac{x^2 - 2x + 3}{x^2(x^2 + 1)}$$

$$\frac{x^2 - 2x + 3}{x^2(x^2 + 1)}$$

$$\frac{x^2 - 2x + 3}{x^2(x^2 + 1)} dx$$
The method of partial fractions
$$\frac{x^2 - 2x + 3}{x^2(x^2 + 1)} = \frac{A}{x^2} + \frac{B}{x} + \frac{Cx + D}{x^2 + 1}$$
Solve for  $A_1 b_1 c$  and integrate.

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#### Integration by parts



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### Elementary functions

#### The elementary functions are

- 26 · Polynomials xn. Actuelly all power of x
- Exponentials  $a^{x}$ .
- Logarithms.
- Trigonometric functions sin x, cos x etc.
- Inverse trigonometric functions,  $\sin^{-1} x$ ,  $\cos^{-1} x$  etc.

And every function that can be built from these by addition, subtraction, multiplication, division and composition.

Ex: 
$$\frac{2}{\frac{\sin x}{\cos x}}$$
 and  $\ln\left(\frac{\tan^2 x}{\infty}\right)$  are elementary

Can we integrate every continuous function?

$$\int e^{x^2} dx \qquad \int \frac{1}{\ln x} dx \qquad \int \frac{\sin x}{x} dx$$

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