Areas

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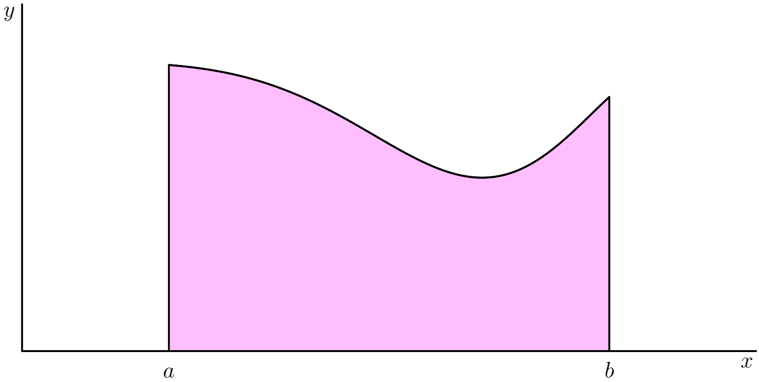
MA100

The area problem



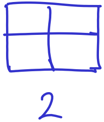
- What is the area of the shaded region?
- What do we *mean* by “the area of the shaded region”?

The area problem (cont.)

For what type of regions can we compute the area? In order of complexity:

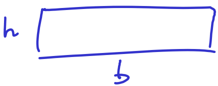
1. Squares
2. Rectangles
3. Triangles
4. Triangulated regions.

1



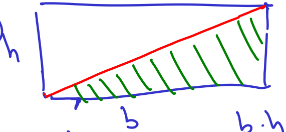
Area 4 sq. units

2



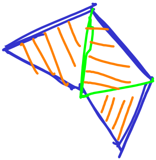
Area
$b \cdot h$ sq units

3



Area of $\triangle$ is $\frac{b \cdot h}{2}$

4



Area: sum of triangle areas.

The area problem (cont.)

Let $n = 4$

(the number of subintervals)

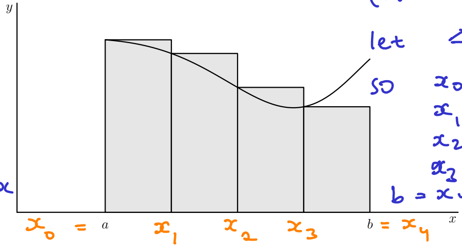
let $\Delta x = \frac{b-a}{n} = \frac{b-a}{4}$

so
$x_0 = a$
$x_1 = a + \Delta x$
$x_2 = a + 2\Delta x$
$x_3 = a + 3\Delta x$
$b = x_4 = a + 4\Delta x$

so $x_i = a + i\Delta x$

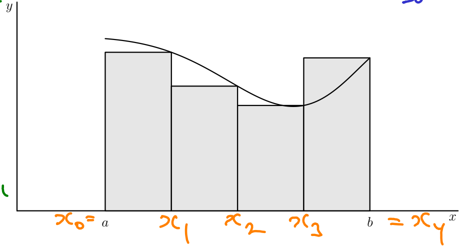
The area of the rectangles in first figure

$= f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$



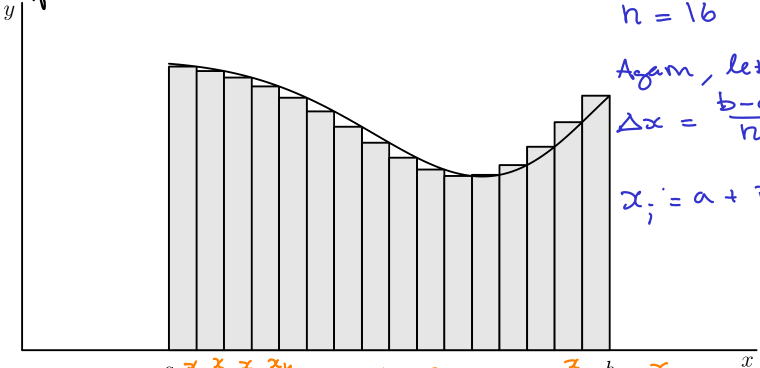
In second figure the rectangles' area is

$f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$



The area problem (cont.)

By increasing n = # subintervals, we get a better approximation of the area



$$x_1 \quad x_2 \quad x_3 \quad x_4 \quad \cdots \quad x_{15} \quad b = x_{16}$$

$n = 16$

Again, let

$$\Delta x = \frac{b-a}{n} = \frac{b-a}{16}$$

$$x_i = a + i\Delta x$$

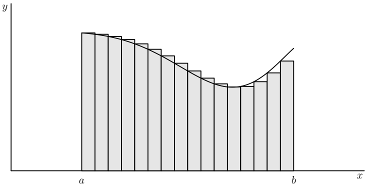
The rectangle area is here =

$$f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + \cdots + f(x_{15})\Delta x + f(x_{16})\Delta x =$$

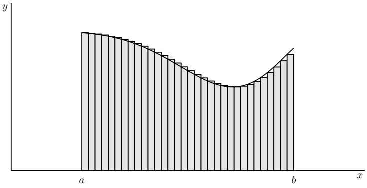
$$= \sum_{i=1}^{16} f(x_i)\Delta x$$

The area problem (cont.)

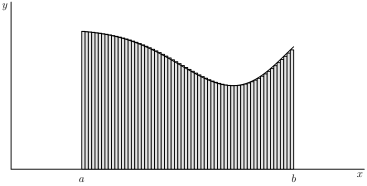
The more intervals, the better the approximation.



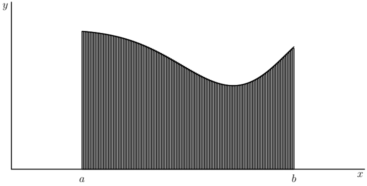
16 subintervals.



32 subintervals



128 subintervals



256 subintervals

Definition of area

Definition

The area A of the region bounded by:

- Below by the x-axis
- From the left by the line $x = a$
- From the rigth by the line $x = b$
- Above by the graph of $f(x)$

where $f(x)$ is a continuous function is the limit of the area of the union of the approximating rectangles

$$A = \lim_{n \to \infty} \left(f(x_1)\Delta_x + f(x_2)\Delta_x f(x_2)\Delta_x + \cdots + f(x_n)\Delta_x \right)$$

Here, $\Delta_x = (b - a)/n$ and $x_i = a +$ [redacted] i [redacted] Δ_x.

Example

The speed of a runner increased steadily during the first three seconds of a race. The speed was measured at each half-second, and the readings are given in the table below.

t(s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v(m/s)	0	1.9	3.3	4.5	5.5	5.9	6.2

Estimate the distance she ran during the first three seconds.

An approximation would be got by assuming that the speed is constant throughout each ½-second interval.

- A lower estimate would be to assume that the speed on each interval is the initial speed in the interval
- An upper estimate would be obtained by instead taking the final speed on interval

Example

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v(m/s)	0	1.9	3.3	4.5	5.5	5.9	6.2

Estimate the distance she ran during the first three seconds.

- For the lower estimate we get

$$0.5 \cdot 0 + 0.5 \cdot 1.9 + 0.5 \cdot 3.3 + 0.5 \cdot 4.5 + 0.5 \cdot 5.5 + 0.5 \cdot 5.9$$

$$= 0.5 \cdot 21.1 = 10.55 \text{ m}$$

- For the upper estimate:

$$0.5 \cdot 1.9 + \dots + 0.5 \cdot 6.2 = 13.65 \text{ m}$$