1st Year Calculus
MA101/MA160

Problem Sheet 1

1. Using Newton's method, find approximations up to the 5 decimal places for the zeros in the interval $[1, 2]$ of the following functions.

$$(a) \quad f(x) = -x^2 + x + 1 \qquad (b) \quad g(x) = x^4 - x - 3$$

2. Use Newton's method to find approximations of the numbers

$$(a) \quad \sqrt[6]{18}, \qquad (b) \quad \sqrt[50]{50},$$

that are correct to six decimal places.

3. Find the most general antiderivative of the functions:

$$(a) \quad f(x) = 1 - x \qquad (b) \quad g(x) = \frac{12}{x^7} \qquad (c) \quad h(x) = \sqrt[3]{x}$$

4. Find, in each case, the function f:
   (a) $f'(x) = 1 - 6x, \quad f(0) = 8.$
   (b) $f'(x) = 2x - 3/x^4, \quad x > 0, \quad f(1) = 3.$
   (c) $f''(\theta) = \sin\theta + \cos\theta, \quad f(0) = 3, \quad f'(0) = 4$

5. Graph the function $f(x) = \cos(x^2)$, $0 \le x \le 1$, and estimate the area below the graph of f using four approximating rectangles and taking the sample points to be
   (a) left endpoints
   (b) right endpoints
   (c) midpoints

   in each case sketch the curve and the rectangles.

6. Evaluate the following integrals by interpreting them in terms of areas:
   (a) $\int_2^4 x\,dx$
   (b) $\int_{-2}^3 |x|\,dx$
   (c) $\int_{-3}^3 \sqrt{9 - x^2}\,dx.$

7. Evaluate the following integrals.
   (a) $\int_{-1}^{1} \left(\frac{2}{5}x^5 - 4x^3 + 1\right)\,dx$
   (b) $\int_0^1 (\sqrt{x} + \cos x)\,dx$
   (c) $\int_0^{\frac{\pi}{4}} \cos 3x\,dx$