The GAP package CompTom

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Chapter 1

The CompTom Package

1.1 Introduction

The notion of the table of marks of a finite group G was introduced by William Burnside in the second edition of his famous book, Theory of Groups of Finite Order, [Bur55], and it is often referred to as the Burnside matrix of G. The table of marks of G is a square matrix whose rows and columns are indexed by the conjugacy classes of subgroups of G and where for any two conjugacy classes of subgroups H and K, the (H,K)-entry is the number of fixed points of K in the transitive action of G on the cosets of H in G. In this way the table of marks of G characterizes the set of all permutation representations of G up to equivalence. At the same time the table of marks of G describes the partially order set of all conjugacy classes of subgroups of G since from the number of fixed points the number of conjugates of K contained in H can be derived.

1.2 Computing Tables of Marks

Traditionally the computation of the table of marks of a finite group G requires complete knowledge of the entire subgroup lattice of G. The marks are then derived by counting inclusions between conjugacy classes of subgroup. This works fine when the order of G is small but is impractical when the order of G is large. Pfeiffer [Pfe97] has developed a method for computing the table of marks of G from the tables of marks of the maximal subgroups of G. This method has been successfully used to compute the tables of marks of a variety of groups including, M_{24}, Mcl, S_{12} and A_{13}. However since this approach relies on the knowledge of the tables of marks of all maximal subgroups of G, the range of simple and almost simple groups to which it can be applied has almost been exhausted. With this in mnd the authors have developed a collection of algorithms to compute the table of marks of a cyclic extension $G.p$ from the table of marks of G. Many groups whose tables of marks have not been computed fall into this category. The functions contained in this package are based on the algorithms described in [NP].

1.3 Installing and Loading the CompTom Package

Download the archives in your preferred format. Unpack the archives inside the pkg dirctory of your GAP installation. Load the package

Example
```
gap> LoadPackage("COMPTOM");
true
```

1.4 Computing Subgroups

Let A be a normal subgroup of S of index p a prime.

1.4.1 SubExtensionsSubgroups

◊ `SubExtensionsSubgroups(S, a, list)` (function)

`SubExtensionSubgroups` takes as input groups `S` and `A`, where `A` is normal in `S` of prime index, and a list of representatives of the conjugacy classes of subgroups of `A` and computes a list of representatives of the conjugacy classes of subgroups of `S`.

Example
```
gap> A := AlternatingGroup(4);S := SymmetricGroup(4);
Alt( [ 1 .. 5 ] )
Sym( [ 1 .. 4 ] )
gap> subs := List(ConjugacyClassesSubgroups(A), Representative);
[ Group(()), Group([ (1,2)(3,4) ]), Group([ (2,4,3) ]),
  Group([ (1,3)(2,4), (1,2)(3,4) ]),
  Group([ (1,3)(2,4), (1,4)(2,3), (2,4,3) ]) ]
gap> SubExtensionsSubgroups(S, A, subs);
[ Group(()), Group([ (1,2)(3,4) ]), Group([ (2,4,3) ]),
  Group([ (1,3)(2,4), (1,2)(3,4) ]),
  Group([ (1,3)(2,4), (1,4)(2,3), (2,4,3) ]), Group([ (1,2) ]),
  Group([ (1,2)(3,4), (3,4) ]), Group([ (1,2)(3,4), (1,4,2,3) ]),
  Group([ (2,4,3), (3,4) ]), Group([ (1,3)(2,4), (1,2)(3,4), (3,4) ]),
  Group([ (2,4,3), (1,4)(2,3), (1,2)(3,4), (3,4) ]) ]
```

1.4.2 AllSubgroupClassesSolvable

◊ `AllSubgroupClassesSolvable(G)` (function)

Given a solvable group `G`, `AllSubgroupClassesSolvable` computes a list of representatives of the conjugacy clases of subgroups of `G`, by first computing a composition series, and then applying `SubExtensionsSubgroups` repeatedly.

Example
```
gap> G := SymmetricGroup(4);
Sym( [ 1 .. 4 ] )
gap> AllSubgroupClassesSolvable(G);
[ Group(()), Group([ (1,2)(3,4) ]), Group([ (1,2)(3,4), (1,4)(2,3) ]),
  Group([ (1,2,3) ]), Group([ (1,2)(3,4), (1,4)(2,3), (2,3,4) ]),
  Group([ (1,2) ]), Group([ (1,2)(3,4), (3,4) ]),
  Group([ (1,2)(3,4), (1,3,2,4) ]), Group([ (1,2)(3,4), (1,4)(2,3), (3,4) ]),
  Group([ (1,2,3), (2,3) ]),
  Group([ (1,4)(2,3), (1,3)(2,4), (2,3,4), (3,4) ]) ]
```

1.5 Computing Tables of Marks

1.5.1 TomExtensionTom

◊ `TomExtensionTom(S, A, tom, maxcoma)` (function)

`TomExtensionTom` computes the table of marks of *S* from the table of marks of *A*. *A* must be normal in *S* of prime index. The third argument *tom* is the table of marks of *A*, while the additional argument *maxcoma*, is a bound on the number of contained maps,.

Example

```
gap>  A := AlternatingGroup(4); S := SymmetricGroup(4);
Alt( [ 1 .. 5 ] )
Sym( [ 1 .. 4 ] )
gap> tom := TableOfMarks(A); maxcoma := 1000;
TableOfMarks( Alt( [ 1 .. 4 ] ) )
1000
gap> TomExtensionTom(S, A, tom, maxcoma);
 TableOfMarks( "S4" )
```

1.5.2 TomSolvableTom

◊ `TomSolvableTom(G, maxcoma)` (function)

Given a solvable group *G* this function comptes the table of marks of *G* in an iterative fashion by first computing a composition series and then applying `TomExtensionTom` repeatedly.

Example

```
gap> G := SymmetricGroup(4);
Sym( [ 1 .. 4 ] )
gap> TomSolvableTom(G, 1000);
 TableOfMarks( "S4" )
```

References

[Bur55] W. Burnside. *Theory of groups of finite order*. Dover Publications Inc., New York, 1955. Unabridged republication of the second edition, published in 1911. 4

[NP] L. Naughton and G. Pfeiffer. Computing the table of marks of a cyclic extension. 4

[Pfe97] G. Pfeiffer. The subgroups of M_{24}, or how to compute the table of marks of a finite group. *Experiment. Math.*, 6(3):247–270, 1997. 4

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