Gauss-Jordan method for calculating a matrix inverse

Task: Calculate the inverse of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix}.$$

Strategy: Write the 3×6 matrix $(A|I)$.
Apply Gauss-Jordan elimination to this matrix.
If A is invertible, the outcome is a 3×6 matrix with I in the first three columns, and the inverse of A in the second three columns.

Goals for this lecture

1. To implement the Gauss-Jordan method
2. To understand why this method produces the inverse of A.

Elementary row operations as matrix products

If A is a matrix with three rows, then

Applying the operation $R2 \to R2 - 2R1$ to A multiplies A on the left by $\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$:

$$\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} R1 \\ R2 \\ R3 \end{pmatrix} = \begin{pmatrix} R1 \\ R2 - 2R1 \\ R3 \end{pmatrix}$$

Every elementary row operation on A is equivalent to multiplying A on the left by an elementary matrix.

Elementary row operations as matrix products

If A is a matrix with three rows, then

Applying the operation $R2 \leftrightarrow R3$ to A multiplies A on the left by $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} R1 \\ R2 \\ R3 \end{pmatrix} = \begin{pmatrix} R1 \\ R3 \\ R2 \end{pmatrix}$$

Every elementary row operation on A is equivalent to multiplying A on the left by an elementary matrix.

Elementary row operations as matrix products

If A is a matrix with three rows, then

Applying the operation $R2 \to 3R2$ to A multiplies A on the left by $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} R1 \\ R2 \\ R3 \end{pmatrix} = \begin{pmatrix} R1 \\ 3R2 \\ R3 \end{pmatrix}$$

Every elementary row operation on A is equivalent to multiplying A on the left by an elementary matrix.