Wednesday 24/10/18 Rachel Quinlan.

Gauss – Jordan:

1. Get a 1 in the upper left.
2. Use this to "clear out" column 1.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 5 & 0 & 1 & 0 \\ 3 & 8 & 6 & 0 & 0 & 1 \end{array}\right]$$

"Done" ← don't add Row 1 to another Row again!

$$\xrightarrow[R3 \to R3 - 3R1]{R2 \to R2 - 2R1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & \boxed{1} & -1 & -2 & 1 & 0 \\ 0 & 2 & -3 & -3 & 0 & 1 \end{array}\right]$$

. Now get a 1 in position 2 of Row 2 ✓

. Use this to clear out the lower part of Col 2.

$$\xrightarrow{R3 \to R3 - 2R2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & \boxed{-1} & 1 & -2 & 1 \end{array}\right]$$

want 1 here

$$\xrightarrow{R3 \times (-1)} \left[\begin{array}{ccc|ccc} \boxed{1} & 2 & 3 & 1 & 0 & 0 \\ 0 & \boxed{1} & -1 & -2 & 1 & 0 \\ 0 & 0 & \boxed{1} & -1 & 2 & -1 \end{array}\right]$$

Now use the leading 1's in Rows 3 and 2 to clear the entries above them.

$$\xrightarrow[R1 \to R1 - 3R3]{R2 \to R2 + R3} \left[\begin{array}{ccc|ccc} 1 & \boxed{2} & 0 & 4 & -6 & 3 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array}\right]$$

$$\xrightarrow{R1 \to R1 - 2R2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 10 & -12 & 5 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array}\right]$$

Conclusion

$$A^{-1} = \begin{bmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{bmatrix}$$

Check

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{bmatrix} \begin{bmatrix} 10 & -12 & 5 \\ -3 & 3 & -1 \\ -1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Each row operation that was applied to A multiplied it on the left by an <u>elementary matrix</u>.

Overall

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix} \longrightarrow \underbrace{E_k \cdots\cdots E_2 E_1}_{E} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 8 & 6 \end{pmatrix}$$

$$[A \mid I] \longrightarrow E[A \mid I]$$

$$\longrightarrow [EA \mid EI]$$

If $EA = I$, then $E = A^{-1}$

so $EI = A^{-1}$