NUI Galway
OÉ Gaillimh

Semester I Examinations 2017-18

Exam Code(s)	2BA1, 2BAJ1, 2BCS1, 2BCW1, 2BDT1, 2BFS1, 2BHR1, 2BIS1 2BLS1, 2BME1, 2BPS1, 1EM1, 1OA1, 2BCT1, 2UPA1, 2BMS1 3BMS2, 2BPT1, 2BS1, 2EH1, 2FM1, 1SWB1
Exam	Second Year
Module	DIFFERENTIAL FORMS
Module Code	MA2286
External Examiner(s)	Prof T. Brady
Internal Examiner(s)	Prof G. Ellis*
Instructions	Attempt as many parts of questions as you wish. Each question has three parts; each part carries 8 marks. Your total score will be capped at 100 marks.
Duration	2 hours
No. of Pages	3 pages (including this cover page)
Discipline	Mathematics
Requirements:	
Release to Library:	No
Other Materials	Non-programmable calculators & maths tables

1.

(a) A fundraising campaign will incur expenditure at a rate of €10 000 per day. Contrubutions are expected to be high during the early stages of the campaign and tail off as the campaign continues. The rate at which contributions are received is modelled by the 1-form

$$w = (-100\,t^2 + 20\,000)\,dt\ .$$

What are the expected net proceeds?

(b) Find a differential 0-form ω on the oriented interval $S = [-\frac{1}{2}, \frac{1}{2}]$ whose derivative is the 1-form

$$d\omega = \frac{4x}{(x-1)^2(x+1)}\,dx\ .$$

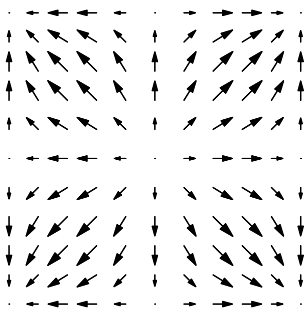
(c) Evaluate the integral

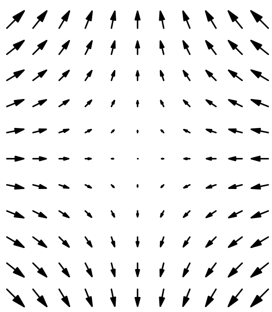
$$\int_S \frac{1}{x(\ln x)^3}\,dx$$

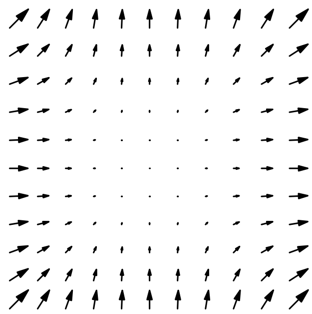
of the differential 1-form $\omega = dx/x(\ln x)^3$ over the oriented interval $S = [e, e^2]$.

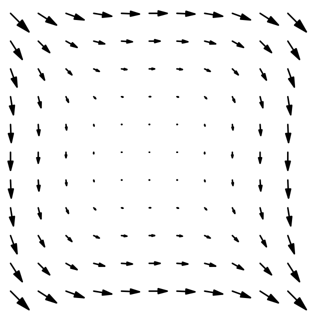
2.

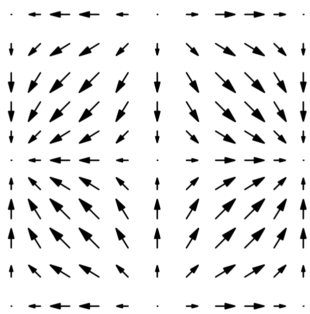
(a) Match the five pictures of flows

(A) 

(B) 

(C) 

(D) 

(E) 

to the five differential 1-forms: (i) $\omega = x^2\,dx + y^2\,dy$, (ii) $\omega = y^2\,dx - x^2\,dy$, (iii) $\omega = -x\,dx + y\,dy$, (iv) $\omega = \sin(\pi x)\,dx + \sin(\pi y)\,dy$, (v) $\omega = \sin(\pi x)\,dx - \sin(\pi y)\,dy$.

(b) Evaluate the integral $\int_C \omega$ of the 1-form

$$\omega = (6yz - 6x^2)\,dx - (3xz + 2y)\,dy + (5xyz^2 - 4)\,dz$$

where C is the curve $x = t$, $y = t^2$, $z = t^3$ from $(-1, 1, -1)$ to $(1, 1, 1)$.

(c) Prove that the 1-form $\omega = (6yz - 6x^2)\,dx - (3xz + 2y)\,dy + (5xyz^2 - 4)\,dz$ does not arise as the total derivative of any differential 0-form.

3.

(a) Let S be a path in the plane from $(1,0)$ to $(2,1)$. Explain why the integral

$$\int_S (1 - ye^{-x})\,dx + e^{-x}\,dy$$

is independent of the choice of path from $(1,0)$ to $(2,1)$, and evaluate this integral.

(b) Evaluate the integral

$$\int_S dy \wedge dz + 2\,dz \wedge dx + 3\,dx \wedge dy$$

where S is the oriented planar triangle with vertices $(2,2,2)$, $(6,10,-2)$, $(8,4,2)$ in that order.

(c) Evaluate the integral

$$\int_S 2\,xy\,dx \wedge dy + x^2 y\,dy \wedge dz$$

where S is the region in xy-plane, with anti-clockwise orientation, bounded by the curves $y = x^2$, $y = \sqrt{2 - x^2}$, $x = 0$ and $x = 1$.

4.

(a) Find the derivatives of the following forms:
(i) $\omega = xy\,dz + yz\,dx + zx\,dy$ (ii) $\omega = x\,dy \wedge dz + y\,dz \wedge dx + z\,dx \wedge dy$

(b) Let S denote the region bounded by the ellipse $x = 3\cos\theta$, $y = 2\sin\theta$ in the xy-plane. Use Stokes' formula to show that the area of S is given by

$$\frac{1}{2}\int_{\partial S} x\,dy - y\,dx.$$

Hence determine the area of S.

(c) Find a unit normal to the surface $S \subset \mathbb{R}^3$ defined by the equation

$$x^2 y - 2xz + 2y^2 z^4 = 10$$

at the pont $(2,1,-1) \in S$.

5.

(a) Let $f(x,y) = \begin{cases} xy/(x^2+y^2)\,, & (x,y) \neq (0,0) \\ 0\,, & \text{otherwise} \end{cases}$. Prove that both $f_x(0,0)$ and $f_y(0,0)$ exist but that $f(x,y)$ is discontinuous at $(0,0)$.

(b) Prove Stokes' formula $\int_{\partial S} \omega = \int_S d\omega$ for $\omega = f(x,y)\colon \mathbb{R}^2 \to \mathbb{R}$ a continuously differentiable function and $S \subset \mathbb{R}^2$ an oriented simple curve with differentiable parametrization $x = g(t)$, $y = h(t)$.

(c) Consider the vector field $F = xz\mathbf{i} - y^2\mathbf{j} + 2x^2y\mathbf{k}$. Define curl$(F)$ in terms of the derivative of a 1-form and then calculate curl(F).