Topology studies properties of spaces or shapes that remain unchanged throughout continuous deformations such as bending and stretching (but not tearing or gluing).

We use toplogy when planning a journey on any metro.

The usual map of the London Underground is a continuous deformation of a geographical map.

The geographical map tells us distances between two stations, whereas the usual deformed map does not. But the deformed map retains enough information for us to plan our journey.

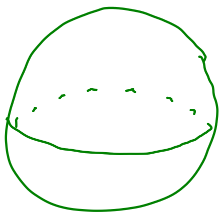
Top ~ places (as in topography)

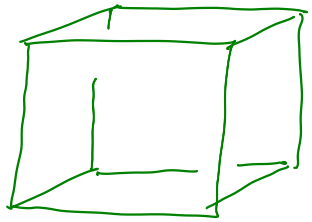
ology ~ study (as in biology)

Motivation for topology:

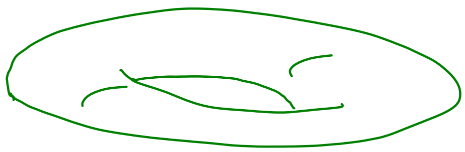
Analysis ~ how best to study and generalise "continuity".

Geometry ~ what properties are common to a hollow sphere and hollow cube





but don't hold for a hollow torus?



A first problem

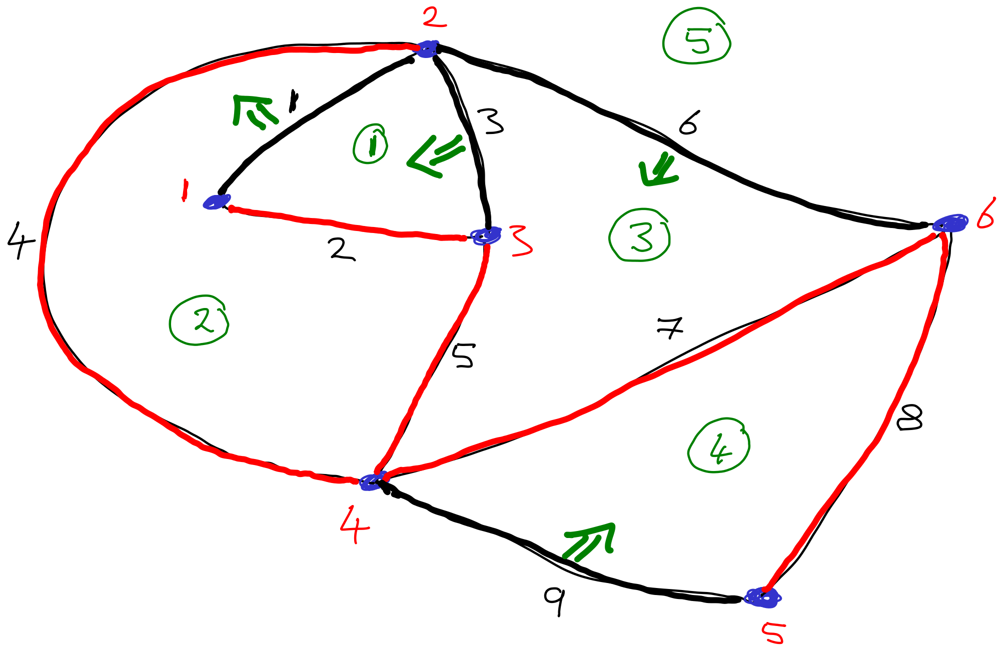
How many pentagons are there in

- a soccer ball
- a carbon-60 molecule
- other potential carbon fullerenes
- a Buckminster dome ?

To tackle this, let's design an arbitrary system of villages and expressways on Mars, subject to:

1) people can drive between any two villages.

2) there are no bridges or tunnels, and expressways meet only at villages,



$V = 6$
$E = 9$
$F = 5$

$$V - E + F = 6 - 9 + 5 = 2$$

Why do we always get $V - E + F = 2$?

Choose a subsystem T of expressways such that:

i) there are no loops

ii) we can still travel between any two villages.

Each black expressway joins two villages in the system T and thus determines a unique loop all but one of whose expressways are red.

The inside of this loop is a field (or collection of fields)

$$V = V_T$$

$$E = E_T + (F-1)$$

$$E_T = V-1.$$

Thus

$$E = V-1 + (F-1) = V+F-2$$

Hence

$$\boxed{\boxed{V - E + F = 2}}$$

Euler's formula

QED

A soccer ball consists of P pentagonal black fields and H hexagonal white fields.

$$V = \frac{5P + 6H}{3}$$

$$E = \frac{5P + 6H}{2}$$

Now

$$2 = V - E + F$$

$$2 = \frac{5P+6H}{3} - \frac{5P+6H}{2} + P + H$$

$$2 = \frac{10P + 12H - 15P - 18H + 6P + 6H}{6}$$

$$2 = \frac{P}{6}$$

Thus $\boxed{\boxed{P = 12}}$

This is true for any sized soccer ball, or fullerene molecule, or dome!