The sphere

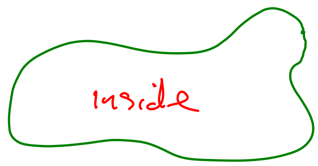
$$S^2 = \{ (x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2 = 1 \}$$

is a more precise notion than the "surface of Mars".

Our explanation of the Euler characteristic formula

$$\chi(S^2) = V - E + F = 2$$

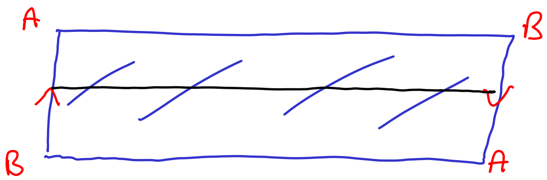
used the fact that any loop on the sphere, with no self intersections



has an inside and an outside. i.e. any such loop cuts the sphere into two pieces.

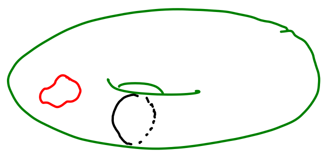
Question : Is this fact "obvious" ?

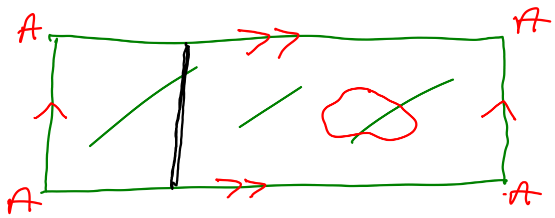
Example Consider the Möbius strip



Draw a loop around the centre of the strip. With pen, paper and scissors check that the black loop does not cut the Möbius strip into two pieces.

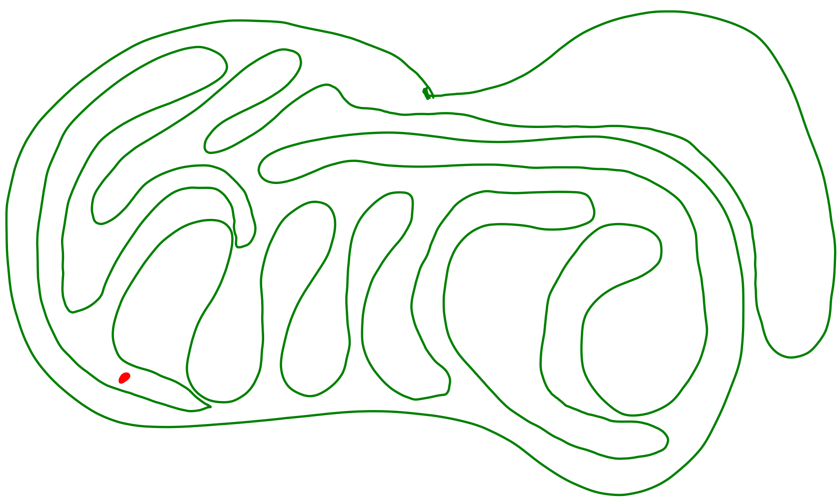
Example Consider the torus





Again, the black loop does not cut the torus into two regions.

Example Is it obvious that the following loop in $\mathbb{R}^2$ cuts the plane into two pieces.



Topology offers a precise language and a collection of techniques for studying such questions.

Let $$S^1 = \{(x,y) \in \mathbb{R}^2 : x^2+y^2=1\}.$$

So S^1 is our notion of circle.

The MA342 module will initially aim at helping us understand the statement of the following result, and also its proof given in Armstrong's book.

Jordan Curve Theorem

Let $\alpha : S^1 \to \mathbb{R}^2$ be any injective continuous function. Let $J \subseteq \mathbb{R}^2$ be the image of α. Then $\mathbb{R}^2 \setminus J$ has two connected components, both of which have frontier J.

Aim for the next few lectures:

1) Explain the above underlined terms.
2) Enable you to read the proof of the theorem in the book.
3) Give some weird examples of α that suggest the theorem is not so obvious.

Definition (Riesz [1909], Hausdorff [1914])

A topological space consists of a set X and a collection T of subsets of X which we call open. The following axioms must hold:

T1) The union of any collection of open sets is open.

T2) The intersection of any finite collection of open sets is open.

T3) Both $\emptyset$ and X are open.

Example 1 $\quad X = \{1, 2, 3, 4\}$

$$T = \{\phi, \{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}, \{1, 2, 3, 4\}\}$$

This is a topological space.