



## Semester 1 Examinations 2018/2019

<b>Exam Codes</b>	1BA1, 1BAJ1, 1BCW1, 1BPS1
<b>Exam</b>	First Arts Examination, First Engineering Examination
<b>Module</b>	Analysis and Algebra 1
<b>Module Code</b>	MA133, MA160-1
<b>Paper No.</b>	1
<b>External Examiner</b>	Prof. Thomas Brady
<b>Internal Examiners</b>	Prof. Graham Ellis Dr John Burns Dr James Cruickshank
<b><u>Instructions</u></b>	<b>Answer all six questions.</b>
<b>Duration</b>	2 hours
<b>No. of Pages</b>	4 pages including this page
<b>School</b>	Mathematics, Statistics & Applied Mathematics
<b><u>Requirements</u></b>	No special requirements
Release to Library:	Yes
Other Materials	Non-programmable calculators Formula and Tables Booklet

**Q1.** (a) Let  $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear transformation with matrix

$$A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$$

with respect to the standard basis vectors.

- (i) [4 marks] Find  $L((1, 0))$  and  $L((0, 6))$ .
- (ii) [4 marks] Find the image of the line  $6x + y = 6$  under the linear transformation  $L$ .

(b) Consider the matrix

$$A = \begin{pmatrix} -3 & 5 \\ -2 & 4 \end{pmatrix}.$$

- (i) [5 marks] Verify that  $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ,  $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$  are eigenvectors of  $A$ . Hence find a diagonal matrix  $D$  and matrix  $E$  such that  $A^n = ED^nE^{-1}$ .
- (ii) [4 marks] Calculate  $A^8$ .

**Q2.** (a) [8 marks] Encipher the plaintext

*H A H A*

by applying the enciphering function

$$f_E: \mathbb{Z}_{26} \rightarrow \mathbb{Z}_{26}, x \rightarrow 14x + 12$$

to 1-letter message units over the alphabet  $A = 0, B = 1, \dots, Z = 25$ . Why is there a problem finding the corresponding deciphering function?

(b) [9 marks] Find the determinant and inverse of the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 3 & 1 & 4 \end{pmatrix}.$$

- Q3.** (a) (i) [3 marks] Use Euclid's Algorithm to find  $\gcd(5, 27)$ .  
(ii) [4 marks] Find  $5^{-1}$  in  $\mathbb{Z}_{27}$ .
- (b) [5 marks] Determine the third digit of the ISBN 3-5?0-90336-4.
- (c) [5 marks]  
Solve the simultaneous congruences

$$x \equiv 4 \pmod{7}$$

$$x \equiv 5 \pmod{8}.$$

- Q4.** (a) Suppose that

$$f(t) = \frac{1-t}{t+1}.$$

- (i) Determine the average rate of change of  $f(t)$  over the interval  $1 \leq t \leq 2$ . [3 marks]  
(ii) Evaluate  $\lim_{t \rightarrow +\infty} f(t)$ . [3 marks]  
(iii) Sketch the graph of  $f$ . [3 marks]
- (b) Evaluate the following limits.
- (i)  $\lim_{h \rightarrow 0} \frac{(2+h)^2 - 4}{h}$ . [4 marks]  
(ii)  $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 2x}$ . [4 marks]

- Q5.** (a) Find the derivative of each of the following functions.

(i)  $f(x) = 3x^2 + 5x - \frac{1}{x}$ . [4 marks]

(ii)  $g(x) = x \cos(x) \sin(x)$ . [4 marks]

(iii)  $h(x) = \sqrt{x^2 + 1}$ . [4 marks]

- (b) Find an equation for the line that is tangent to the curve

$$y = \frac{1}{x^2 + 1}$$

at the point  $(-1, \frac{1}{2})$ . [5 marks]

- Q6.** (a) The function  $f(x) = x + \frac{4}{x}$  has 2 critical points. Find them both. [5 marks]
- (b) Find the maximum and minimum values of  $x + \frac{4}{x}$  for  $1 \leq x \leq 3$ . [5 marks]
- (c) A box is to be made from a rectangular sheet of cardboard that is  $30\text{cm} \times 30\text{cm}$ , by cutting equal squares out of the four corners and bending up the resulting flaps to make the sides of the box (the box has no top). What is the largest possible volume of such a box? [7 marks]