MA313 Linear Algebra

Video 1: Welcome

Tobias Rossmann

NUI Galway

Module coordinates

Lecturer: Tobias Rossmann (Email: tobias.rossmann@nuigalway.ie)

Material: pre-recorded videos will be released every week ahead of the scheduled hours:

- Tuesday 1–2pm
- Friday 12 noon–1pm

Tutorials (weeks 2–12): Tuesday 1-2pm in AC202 (TBC!)

Book: D. C. Lay: “Linear Algebra and Its Applications”, 4th ed.

Assessment:

- 50% final exam: 2 hour written exam
- 30% continuous assessment (WeBWorK)
- 20% communication skills (jointly with MA335)

Communication skills?

You will be asked to write a short essay and give a short (virtual) presentation. Further details will be provided later. This will be a joint effort between MA313 Linear Algebra and MA335 Algebraic Structures

Questions?

- The Discussion Board on your Blackboard page is the preferred method of communication. You are welcome to post messages at any time.
- Any questions posted will be answered during the scheduled lecture times.
- If you wish to contact me privately, please send me an email. Please check the discussion board first to see if your question has already been answered. Please also include the module code in the subject line of your email.
- For further details, please see the "Information" section on Blackboard.

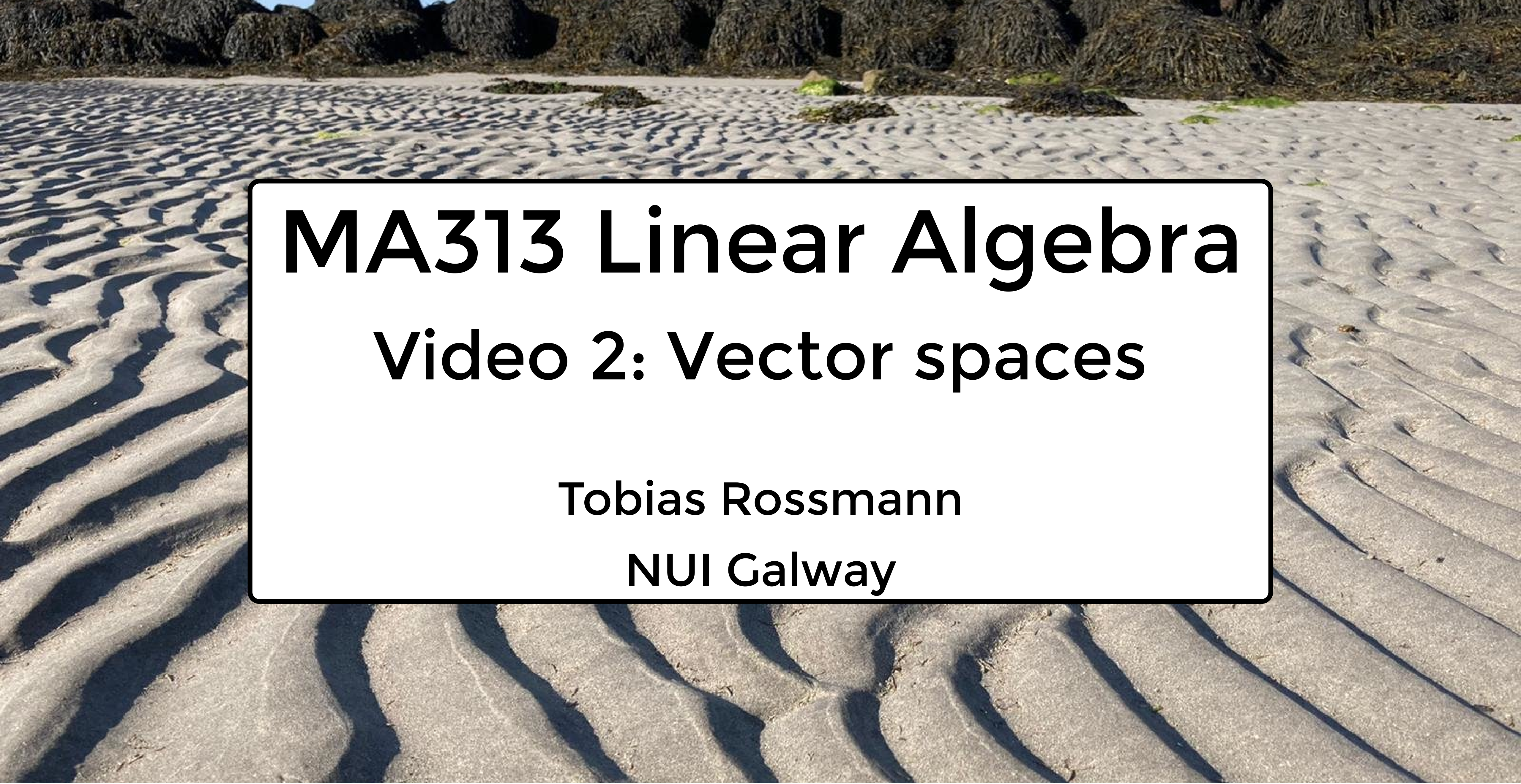
Module content

We'll study

- vector spaces,
- linear transformations, and
- orthogonality,

and we'll see how these topics can be applied to

- signal processing,
- computer graphics, and
- data fitting.



MA313 Linear Algebra

Video 2: Vector spaces

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Recap: previous modules on linear algebra

MA133

$\mathbb{R}^2$ and 2×2 matrices:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

MA203

$\mathbb{R}^n$, determinants, eigenvalues, row reduction, ...

What's left?

"Definition"

Informally, a **vector space** is a collection of *“things”* (which we call **vectors**) which can be “manipulated like” vectors in some $\mathbb{R}^n$... but without coming with intrinsic *“coordinates”.*

Why bother?

- Important examples of vector spaces don't come with "natural" coordinates.

  Example: function spaces which are e.g. used to represent signals
- Abstract vector spaces cast new light on $\mathbb{R}^n$.

  Example: what happens if you change coordinates?
- The theory of vector spaces illustrates the **development of mathematical theories** and the role of **abstraction** in mathematics

Getting formal

Definition

A **vector space** consists of

- a (non-empty!) set V whose elements we call **vectors**,
- an operation called **addition** which assigns a vector
$$u + v \in V$$
to any two vectors $u, v \in V$, and
- an operation called **scalar multiplication** which assigns a vector
$$cu \in V$$
to each scalar $c \in \mathbb{R}$ and vector $u \in V$

such that the axioms on the following slides are satisfied.

Definition (cont.)

We require that the following conditions **V1–V8** are satisfied for all vectors $u, v, w \in V$ and scalars $c, d \in \mathbb{R}$:

V1. $u + v = v + u$ (**commutativity** of addition)

V2. $(u + v) + w = u + (v + w)$ (**associativity** of addition)

V3. There exists $0 \in V$, called the **zero vector,** such that for all $u \in V$,

$$u + 0 = u.$$

V4. For each $u \in V$, there exists $-u \in V$ such that $u + (-u) = 0$.

Definition (cont.)

V5. $c(u+v) = cu + cv$ (**distributivity I**)

V6. $(c+d)u = cu + du$ (**distributivity II**)

V7. $c(du) = (cd)u$

V8. $1u = u.$

Theory...

Vector spaces

Linear transformations

Orthogonality

… and applications

Signal processing:

real-life signals $\longrightarrow$ "discrete signals" $\longrightarrow$ output

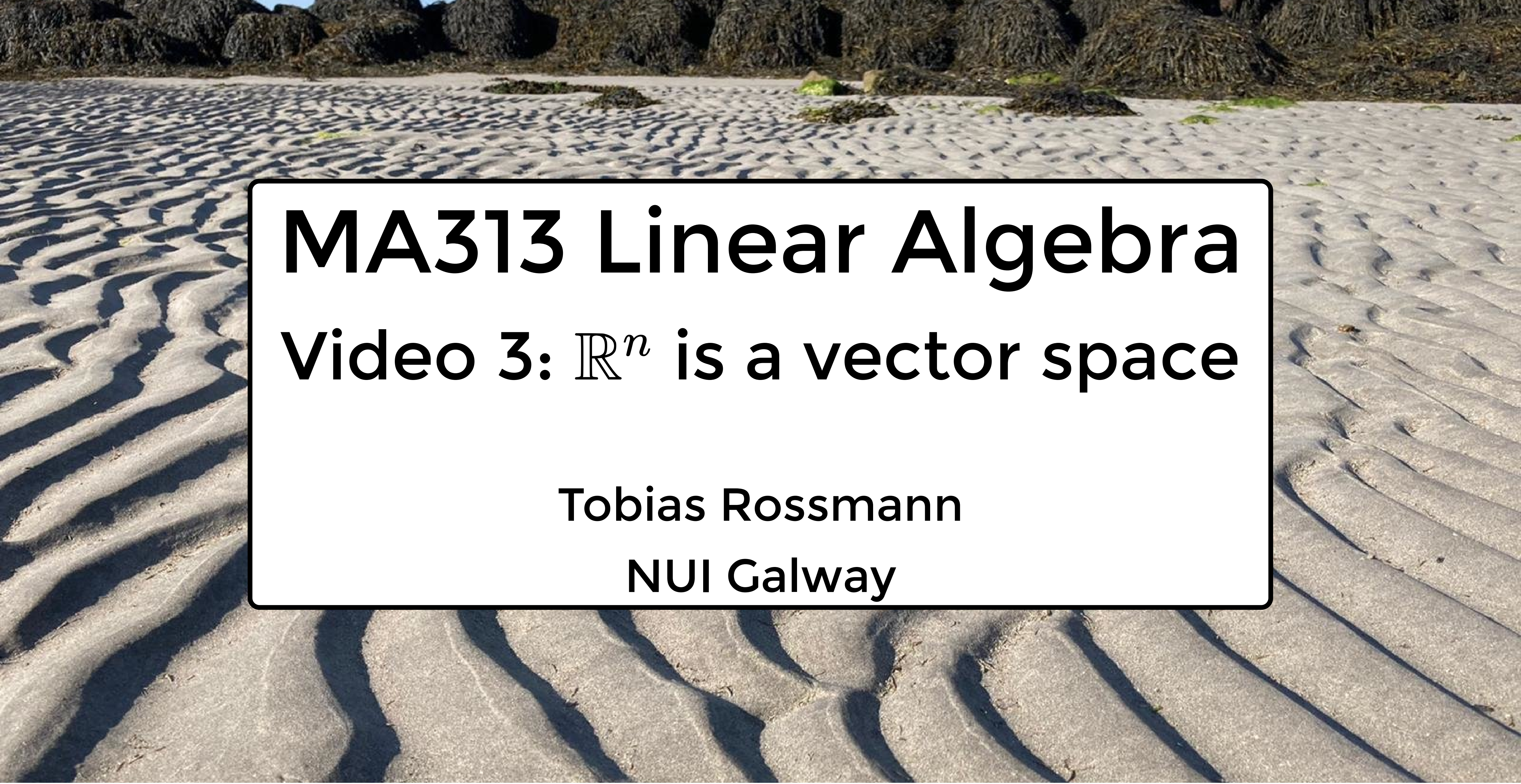
"quantisation"

"filter" (e.g. equaliser)

Computer graphics (e.g. movements or projections)

Data fitting:

match "noisy" imprecise real-world data and a mathematical model



MA313 Linear Algebra

Video 3: $\mathbb{R}^n$ is a vector space

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Example

$$\mathbb{R}^n = \left\{ \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} : x_1, \ldots, x_n \in \mathbb{R} \right\}$$ with addition

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} := \begin{bmatrix} x_1 + y_1 \\ \vdots \\ x_n + y_n \end{bmatrix}$$

and scalar multiplication

$$c \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} := \begin{bmatrix} cx_1 \\ \vdots \\ cx_n \end{bmatrix}$$

is a vector space. The proof is a bit tedious but easy!