

## CS319: Scientific Computing (with C++)

Niall Madden (Niall.Madden@NUIGalway.ie)

## Week 8: Linear Systems, and Operator Overloading

9am, 30 March, and 4pm, 31 March, 2021

### 1 Part 1: Solving Linear Systems (again)

- Jacobi's method
- Implementation

### 2 Part 2: A matrix class

- MatVec

### 3 Part 3: Coding Jacobi's method

### 4 Part 4: Copy Constructors

- A new constructor

### 5 Part 5: Operator Overloading

- Eg 1: Adding two vectors

### 6 Part 6: The `->`, `this`, and `=` operators

- The `->` operator
- The `this` pointer
- Overloading `=`

Tuesday

See "extras" section of today's lectures for more examples of classes and overloading (points, dates, complex numbers); Code for these is in the [Week08/extras/](#) folder on the repository/website.

*These slides do not include all issues concerning operator overloading. Among the topics omitted are:*

- ▶ overloading the unary `++` and `--` operators. There are complications because they work in both prefix and postfix form.
- ▶ Overloading the ternary operator: `? :`
- ▶ **Important:** overloading the `[]` operator.

wed

## Usual reminders...

	Mon	Tue	Wed	Thu	Fri
9 – 10		LECTURE	X		
10 – 11		LAB			
11 – 12					
12 – 1					
1 – 2		LAB			
2 – 3					
3 – 4					
4 – 5			LECTURE		

1. Two recorded classes this week: Tuesday at 09.00, and Wednesday at 16.00.
2. **Lab times: Tuesday 10.00-10.50, and 13.00-13.50.** You should try to attend at least one of these.

CS319 – Week 8  
Week 8: Linear Systems, and Operator Overloading

Start of ...

# PART 1: Solving Linear Systems (again)

This continues from where we left off in Week 7

continuing  
from Part 4  
of Week 7.

## Part 1: Solving Linear Systems (again)

Our eventual goal is the solve systems of  $N$  simultaneous equations in  $N$  unknowns: *find*  $x_1, x_2, \dots, x_N$ , *such that*

$$\begin{array}{l} \text{Eg} \\ 6x_1 + 3x_2 - x_3 = 10 \\ 2x_1 + 7x_2 + x_3 = 1 \\ -x_1 - x_2 + 4x_3 = -5 \end{array} \left| \begin{array}{l} a_{11}x_1 + a_{12}x_2 + \dots + a_{1N}x_N = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2N}x_N = b_2 \\ \vdots \\ a_{N1}x_1 + a_{N2}x_2 + \dots + a_{NN}x_N = b_N. \end{array} \right.$$

We expressed this as a matrix-vector equation: *Find*  $\mathbf{x}$  *such that*

$$\mathbf{Ax} = \mathbf{b}$$

where  $A$  is a  $N \times N$  matrix, and  $\mathbf{b}$  and  $\mathbf{x}$  are (column) vector with  $N$  entries.

We could do this with **Gaussian Elimination** (or  $LU$ -factorization, etc). But instead we use a method that is easier to program: **Jacobi's method**.

$$\begin{pmatrix} 6 & 3 & -1 \\ 2 & 7 & 1 \\ -1 & -1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 1 \\ -5 \end{pmatrix}$$

The idea is to choose an initial “guess”, which call  $\mathbf{x}^{(0)}$ .

Then we try to compute an improved guess, called  $\mathbf{x}^{(1)}$ .

And we improve that again, to get  $\mathbf{x}^{(2)}$ .

Eventually, we have a sequence of estimates

$$\{\mathbf{x}^{(0)}, \mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \mathbf{x}^{(3)}, \dots, \mathbf{x}^{(k)}, \dots\}$$

If we could do this an infinite number of times, then

$$\text{as } k \rightarrow \infty, \text{ we get } \mathbf{x}^{(k)} \rightarrow \mathbf{x}.$$

But in practice, we just iterate until  $\mathbf{x}^{(k)}$  is “close enough” to  $\mathbf{x}$ .

$$\mathbf{x}^{(0)} = \begin{pmatrix} x_1^{(0)} \\ x_2^{(0)} \\ x_3^{(0)} \end{pmatrix}$$

The algorithm (i.e., the method of “improving” the  $\mathbf{x}^{(k)}$ ) comes from the observation that, since (for example)

The 1<sup>st</sup> Eqn  $\rightarrow a_{11}x_1 + a_{12}x_2 + \dots + a_{1N}x_N = b_1,$

then

$$x_1 = \frac{1}{a_{11}}(b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1N}x_N)$$

*Rearranged*

So we can be optimistic that if  $x_2^{(k)}, x_3^{(k)}, \dots, x_N^{(k)}$  are a good estimates for  $x_2, x_3, \dots, x_N$ , then

$$x_1^{(k+1)} = \frac{1}{a_{11}}(b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)} - \dots - a_{1N}x_N^{(k)})$$

will be an even better one for  $x_1$ .

Here  $k = 0, 1, 2, 3, 4, \dots$

Applying the same idea to the rest of the equations, we get

$$\begin{aligned}
 x_1^{(k+1)} &= \frac{1}{a_{11}} (b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)} - \dots - a_{1N}x_N^{(k)}) \\
 x_2^{(k+1)} &= \frac{1}{a_{22}} (b_2 - \underbrace{a_{21}x_1^{(k)}} - \underbrace{a_{23}x_3^{(k)}} - \dots - \underbrace{a_{2N}x_N^{(k)}}) \\
 &\vdots \\
 x_N^{(k+1)} &= \frac{1}{a_{NN}} (b_N - a_{N,1}x_1^{(k)} - \dots - a_{N,N-1}x_{N-1}^{(k)})
 \end{aligned}$$

This can be programmed with two (or so) nested `for` loops (which is the topic of Lab 6). But it can also be expressed in a simple way, using matrices and vectors.

Example

See video or annotated slides

$$6x_1 + 3x_2 - x_3 = 10$$

$$2x_1 + 7x_2 + x_3 = 1$$

$$-x_1 - x_2 + 4x_3 = -5$$

Take  $x^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$\text{So } x_1^{(1)} = \frac{1}{6} (10 - 3x_2^{(0)} + x_3^{(0)}) = \frac{10}{6}$$

$$x_2^{(1)} = \frac{1}{7} (1 - 2x_1^{(0)} - x_3^{(0)}) = \frac{1}{7}$$

$$x_3^{(1)} = \frac{1}{4} (-5 + x_1^{(0)} + x_2^{(0)}) = -\frac{5}{4}$$

$$\text{So } x^{(1)} = \begin{pmatrix} 10/6 \\ 1/7 \\ -5/4 \end{pmatrix}$$



Take  $x^{(1)} = \begin{pmatrix} 10/6 \\ 1/7 \\ -5/4 \end{pmatrix}$

See video or annotated slides

So  $x_1^{(2)} = \frac{1}{6} (10 - 3x_2^{(1)} + x_3^{(1)}) =$

$$\frac{1}{6} (10 - 3(\frac{1}{7}) - \frac{5}{4}) = 1.3869$$

Similarly compute  $x_2^{(2)}$  and  $x_3^{(2)}$

---

There is also a "Matrix View" of this.

See video or annotated slides

The initial system is  $Ax = b$ .

We can write the Jacobi iteration in matrix form too.

$$\text{Let } D = \text{diag}(A) = \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix} \quad \text{Note: } D^{-1}$$

$$\text{Let } T = D - A = \begin{pmatrix} 0 & -a_{12} & -a_{13} \\ -a_{21} & 0 & -a_{23} \\ -a_{31} & -a_{32} & 0 \end{pmatrix} \quad \left. \begin{matrix} \\ \\ \\ \end{matrix} \right\} \begin{pmatrix} 1/a_{11} & 0 & 0 \\ 0 & 1/a_{22} & 0 \\ 0 & 0 & 1/a_{33} \end{pmatrix}$$

The Jacobi iteration is

$$x^{(k+1)} = D^{-1} (b + Tx^{(k)})$$

Now that we know the method, let us summarise the steps, so as to work out what standard operations on vectors and matrices we need.

We expressed the problem as a matrix-vector equation: *Find  $\mathbf{x}$  such that*

$$A\mathbf{x} = \mathbf{b},$$

where  $A$  is a  $N \times N$  matrix, and  $\mathbf{b}$  and  $\mathbf{x}$  are (column) vector with  $N$  entries.

We then derived **Jacobi's method**: choose  $\mathbf{x}^{(0)}$  and set

$$\mathbf{x}^{(k+1)} = D^{-1}(\mathbf{b} + T\mathbf{x}^{(k)}).$$

where  $D = \text{diag}(A)$  and  $T = D - A$ .

Looking at this we see that the fundamental operations are: **vector addition** and **matrix-vector multiplication.**

**CS319 – Week 8**  
**Week 8: Linear Systems, and Operator Overloading**

**END OF PART 1**

CS319 – Week 8  
Week 8: Linear Systems, and Operator Overloading

Start of ...

**PART 2: A matrix class**

## Part 2: A matrix class

Since we already have `Vector` class from last week, our next step is to write a `class` implementation for a `matrix`, along with the associated functions.

Then we need to define a `function to multiply a matrix by vector`.

First though, we consider the matrix representation. The most natural approach might seem to be to construct a two dimensional array. This can be done as follows (see Lab 4):

```
double **entries = new double *[N];
for (int i=0; i<N; i++)
    entries[i] = new double N;
```

*An array of arrays.*

A simpler, faster approach is to store the  $N^2$  entries of the matrix in a single, one-dimensional, array of length  $N^2$ , and then take care how the access is done:

In C++, if we want, we can just represent a matrix  $A$  as a "big vector" programmed as

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

represented as

$$\begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{31} \\ a_{32} \\ a_{33} \end{pmatrix}$$

## Part 2: A matrix class

This weeks files also  
have an updated "  
Matrix08.h".

### Matrix.h

```
1 // File: Matrix.h (W07.1)
2 // Author: Niall Madden (NUI Galway) Niall.Madden@NUIGalway.ie
3 // Date: Week of 2021-CS319)
4 // What: Implementation of "Matrix": a class of square matrices
5 // See also: Matrix.cpp and O2TestMatrix.cpp

6
7 class Matrix {
8 private:
9     double *entries;
10    unsigned int N;
11 public:
12     Matrix(unsigned int Size=2);
13     ~Matrix(void) { delete [] entries; };
14
15     unsigned int size(void) {return (N);};
16     double getij (unsigned int i, unsigned int j);
17     void setij (unsigned int i, unsigned int j, double x);
18
19     void print(void);
20 };
```

(Compared with Vector.h)

} → where we store values  
} → number of rows and cols

} constructor +  
destructor.

→ For returning an  
entry in  
the matrix

## Part 2: A matrix class

from Matrix.cpp

```
10 Matrix::Matrix (unsigned int Size)
11 {
12     N = Size;
13     entries = new double [N*N];
14 }
15
16 void Matrix::setij (unsigned int i, unsigned int j, double x)
17 {
18     if (i<N && j<N)
19         entries [i*N+j] = x;
20     else
21         std::cerr << "Matrix::setij(): Index out of bounds."
22                 << std::endl;
23 }
```

} constructor.

Just check that  $i$  &  $j$  are valid (ie, in the range  $0, \dots, N-1$ )



## Part 2: A matrix class

from Matrix.cpp

```
24 double Matrix::getij (unsigned int i, unsigned int j)
25 {
26     if (i<N && j<N)
27         return entries[i*N+j];
28     else
29     {
30         std::cerr << "Matrix::getij(): Index out of bounds."
31                 << std::endl;
32         return (0);
33     }
34 }

36 void Matrix::print (void)
37 {
38     // std::cout << "Matrix is of size " << M << "-by-"
39     // << N << std::endl;
40     for (unsigned int i=0; i<N; i++)
41     {
42         for (unsigned int j=0; j<N; j++)
43             std::cout << "[" << entries[i*N+j] << "];"
44             std::cout << std::endl;
45     }
```

So  $a_{ij}$  is stored  
in  $\text{entries}[i*N+j]$ :

We'll test this by implementing matrix-vector multiplication function:

### O2TestMatrix.cpp

```

2 // File:      01TestMatrix.h (Set v=A*u)
  // Author:    Niall Madden (Niall.Madden@NUIGalway.ie)
  // Date:      Week 8 of 2021-CS319)
4 // What:      Test the implementation Matrix class

```

```

48 void MatVec(Matrix &A, Vector &u, Vector &v)
   {
50     unsigned int N;
     N = A.size();
52     if ( (N != u.size()) || ( N != v.size() ) )
         std::cerr << "dimension mismatch in MatVec" << std::endl;
54     else
         for (unsigned int i=0; i<N; i++)
56         {
             double x=0;
58             for (unsigned int j=0; j<N; j++)
                 x += A.getij(i,j)*u.geti(j);
60             v.seti(i,x);
           }
62 }

```

MatVec is  
a Matrix-Vector  
Product.

**CS319 – Week 8**  
**Week 8: Linear Systems, and Operator Overloading**

**END OF PART 2**

CS319 – Week 8  
Week 8: Linear Systems, and Operator Overloading

Start of ...

**PART 3:** Coding Jacobi's Method

Recorded Tue, 30 March (~ 9:45).

## Part 3: Coding Jacobi's method

Now we can implement Jacobi's method. The specific example coded, we will solve  $N = 3$  equations whose matrix representation is

$$9x_1 + 3x_2 + 3x_3 = 15 \quad (1)$$

$$3x_1 + 9x_2 + 3x_3 = 15 \quad (2)$$

$$3x_1 + 3x_2 + 9x_3 = 15 \quad (3)$$

This problem is constructed so that the solution is  $x_1 = x_2 = x_3 = 1$ .

Have a look at the `main()` function in `02Jacobi.cpp` to see how the problem is set up, and how the Jacobi solver is called. Here we will focus on that solver.

$$\text{Soln : } x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
$$A = \begin{pmatrix} 9 & 3 & 3 \\ 3 & 9 & 3 \\ 3 & 3 & 9 \end{pmatrix}$$

$$b = \begin{pmatrix} 15 \\ 15 \\ 15 \end{pmatrix}$$

## Part 3: Coding Jacobi's method

See 02Jacobi.cpp for more details

```
100 // Use Jacobi's method to solve  $Ax=b$ , — i.e.  $x^{(0)}$   
    // On entry : x is the initial guess —  $x^{(k)}$   
102 // On exit : x is the estimate for the solution  
void Jacobi(Matrix &A, Vector &b, Vector &x,  
104             unsigned int &count, double tol)  
{  
106     unsigned int N=A.size();  
    count=0;  
108     if ( (N != b.size()) || (N != x.size()) )  
        std::cout << "Jacobi: error - A must be the same size as b,x"  
110         << std::endl;
```

*making sure A, b & x all have  
the same N.*

## Part 3: Coding Jacobi's method

See 02Jacobi.cpp for more details

```
112 Matrix Dinv(N), T(N); // The diagonal and off-diagonal matrices
    for (unsigned int i=0; i<N; i++)
114     for (unsigned int j=0; j<N; j++)
        if (j != i) (not diagonal)
116         {
            T.setij(i,j, -A.getij(i,j));
118             Dinv.setij(i,j, 0.0);
        }
        else (ie j==i, so diagonal
120             entries)
            {
122                 T.setij(i,j, 0.0);
124                 Dinv.setij(i,j, 1.0/A.getij(i,j));
            }
```

Setting  
up  
Dinv  
& T.

$D_{inv} \sim D^{-1}$  where  $D = \text{diag}(A)$

$T = D - A$

### Part 3: Coding Jacobi's method

$$x^{(k+1)} = D^{-1} (b + T x^{(k)})$$

See 02Jacobi.cpp for more details

```
126 // Now implement the algorithm:
    Vector d(N), r(N);
128 do
    {
130     count++;
    - MatVec(T,x,d); // Set d=T*x
132     - VecAdd(d, b, d); // set d=b+d (so d=b+T*x)
    -> MatVec(Dinv, d, x); // set x = inverse(D)*(b+T*x)

    MatVec(A, x, r); // set r=A*x
136     VecAdd(r, b, r, 1.0, -1.0); // set r=b-A*x

138 } while ( r.norm() > tol);
```

This is  
the  
actual  
Jacobi  
Method.

Since  $Ax = b$ , we know  $b - Ax = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$   
So if  $b - Ax^{(k)}$  is "small", we have a good approximation

Of course, the above code would be a lot neater, and much more readable, if we were able to write, for example,  $r=A*x$  instead of `MatVec(A,x,r)` ....



## Exercise (8.1)

Write a method `Matrix::norm()` that returns the “Entry-wise” 2-norm of a matrix (also called the **Frobenius** or Hilbert–Schmidt norm) :

$$\|A\|_p = \left( \sum_{i=1}^n \sum_{j=1}^n |A_{i,j}|^p \right)^{1/p}.$$

and the max-norm:

$$\|A\|_0 = \max_{i,j} |A_{i,j}|.$$

End of port 3.

Wed, 31 March, 2021

CS319 – Week 8

Week 8: Linear Systems, and Operator Overloading

Start of ...

## PART 4: Copy Constructors

Toward : operator overloading

In the next section, we will introduce the idea of **Operator Overloading**. But to get this to work, we need to study **copy constructors**.

This is a very technical area of C++ programming, but is unavoidable.

As we already know, **constructor** is a method associated with a class that is called automatically whenever an object of that class is declared.

But there are <sup>times</sup> ~~time~~ when objects are *implicitly* declared, such as when passed (by value) to a function.

Since this will happen often, we need to write special constructors to handle it.

method that has the same name  
as the class.

Last week we defined a class for vectors:

- ▶ It stores a vector of  $N$  doubles in a dynamically assigned array called *entries*;
- ▶ The constructor takes care of the memory allocation.

```

2 // From Vector.h (Week 7)
3 class Vector {
4     private:
5         double *entries;
6         unsigned int N;
7     public:
8         Vector (unsigned int Size=2); → constructor
9         ~Vector(void);
10
11         unsigned int size(void) {return N;};
12         double geti (unsigned int i);
13         void seti (unsigned int i, double x);
14         // print(), zero() and norm() not shown
15 };
16
17 // Code for the constructor from Vector.cpp
18 Vector::Vector (unsigned int Size) {
19     N = Size;
20     entries = new double[Size];
21 }

```

Here "entries" is pointer to type double - stores memory address of the start of the array

We then wrote some functions that manipulate vectors, such as `AddVec` in `Week07/01TestVector.cpp`

```
1 void VecAdd (Vector &c, Vector &a, Vector &b,
2             double alpha=1.0, double beta=1.0);
```

— set  $c = \text{alpha} * a + \text{beta} * b$

Note that the `Vector` arguments are passed by reference...

- that is
- ① `&` before the variable name.
  - ② the function get the actual variable's address, not just a copy

What would happen if we tried the following, seemingly reasonable piece of code?

```
Vector a(4);
a.zero(); // sets entries of a all to 0
Vector c=a; // should define a new vector, with a copy of a
```

This will cause problems for the following reasons:

We would like to think it sets

$$c.entries[0] = a.entries[0]$$

$$c.entries[1] = a.entries[1]$$

$$c.entries[2] = a.entries[2]$$

$$c.entries[3] = a.entries[3]$$

But actually, it sets

$$c.entries = a.entries.$$

That is, they both point to some memory address.

This is a problem

① changing  $c.entries[k]$  changes  
 $a.entries[k]$   
and vice versa.

② If  $c$  (for example) goes out of scope, its destructor is called, deleting the memory allocated  $c.entries$ . So  $a.entries$  gets deleted too!

③ When  $a$  goes out of scope,  $a.entries$  is deleted (again!). which causes program to crash.  
**Double delete**

To solve this problem, we should define our own **copy constructor**. A **copy constructor** is used to make an exact copy of an existing object. Therefore, it takes a single parameter: the address of the object to copy. For example:

See Vector08.cpp for more details

```
20 // copy constructor (Version W08.1)
21 // Class definition in Vector08.h has also changed
22 Vector::Vector (const Vector &old_Vector)
23 {
24     N = old_Vector.N;
25     entries = new double[N];
26     for (unsigned int i=0; i<N; i++)
27         entries[i] = old_Vector.entries[i];
28 }
```

*old-Vector not changed.*

*argument is an object of the some class.*

*notice we don't have to give the object's name - it is implicit.*



The **copy constructor** can be called two ways:

(a) *explicitly*, .e.g,

```
Vector V(2);  
V.seti(0)=1.0; V.seti(1)=2.0;  
Vector W(V); // W is a copy V
```

(b) *implicitly*, when ever an object is passed by value to a function. If we have not defined our own copy constructor, the default one is used, which usually causes trouble.

CS319 – Week 8  
Week 8: Linear Systems, and Operator Overloading

END OF PART Part 4

const is a modifier to a argument  
entry in a function that asserts  
that that argument won't be changed.

CS319 – Week 8

Week 8: Linear Systems, and Operator Overloading

Start of ...

**PART 5: Operator Overloading**

Recall that, along with **Encapsulation** (Classes) and **Inheritance** (deriving new classes from old), **Polymorphism** is one of the pillar ideas of Object-Oriented Programming

So far we have seen two forms of **Polymorphism**:

- (a) we may have two functions with the same name but different argument lists. This is **function overloading**.
- (b) **templates** allow us to define a function or class that with arbitrary data-types, which are not specified until used. In the case of a **class template** it is specified when an object of that class is defined.

We'll cover another form of polymorphism today: **"Operator overloading"**.

Our main goal is to overload the addition (+) and subtraction (-) operators for vectors.

*Yesterday's  
Review*

Last week, we wrote a function to add two **Vectors**: **AddVec**.

It is called as **AddVec(c, a, b)**, and adds the contents of vectors *a* and *b*, and stores the result in *c*.

It would be much more natural to redefine the standard **addition** and **assignment** operators so that we could just write **c=a+b**. This is called **operator overloading**.

- To overload an operator we create an **operator function** – usually as a member of the class. (It is also possible to declare an operator function to be a **friend** of a class – it is not a member but does have access to private members of the class. More about **friends** later).

The general form of the operator function is:

```
return-type class-name :: operator # (arguments)  
{  
    : // operations to be performed.  
};
```

*return-type* of a operator is usually the class for which it is defined, but it can be any type.

Note that we have a new key-word: operator. The operator being overloaded is substituted for #

Eg

+ or =

Almost all C++ operators can be overloaded:

+	-	*	/	%	^	&		~	!
=	<	>	+=	-=	*=	/=	%=	^=	&=
=	<<	>>	>>=	<<=	==	!=	<=	>=	&&
	++	--	->*	,	->	[]	()	new	delete

but not . :: .\* ?

Note that, for example, - (minus)

can be both

- binary, as in  $c = a - b$

- unary, as in  $c = -a$ .

Also ++ can be a++ or ++a.

- ▶ Operator precedence cannot be changed:  $*$  is still evaluated before  $+$
- ▶ The number of arguments that the operator takes cannot be changed, e.g., the  $++$  operator will still take a single argument, and the  $/$  operator will still take two. (Eg if  $a$  &  $b$  are ints,  $a + b$  is what it always way.
- ▶ The original meaning of an operator is not changed; its functionality is extended. It follows from this that operator overloading is always relative to a user-defined type (in our examples, a `class`), and not a built-in type such as `int` or `char`.
- ▶ Operator overloading is always relative to a user-defined type (in our examples, a `class`).
- ▶ The assignment operator,  $=$ , is automatically overloaded, but in a way that usually fails except for very simple classes. (some as copy constructor)

$\rightarrow Z = A * B + C$  is  $Z = (A * B) + C$   
 and not  $Z = A * (B + C)$ .



We are free to have the overloaded operator perform any operation we wish, but it is good practice to relate it to a task based on the traditional meaning of the operator. E.g., if we wanted to use an operator to add two matrices, it makes more sense to use `+` as the operator rather than, say, `*`.

We will concentrate mainly on binary operators, but later we will also look at overloading the unary “minus” operator.

.....

For our first example, we’ll see how to overload `operator+` to add two objects from our `vector` class.

First we'll add the declaration of the operator to the class definition in the header file, `Vector08.h`:

```
Vector operator+(Vector b);
```

Then to `Vector08.cpp`, we add the code

return type

See `Vector08.cpp` for more details

```

95 // Overload the + operator.
96 Vector (Vector) : operator+(Vector b)
97 {
98     Vector c(N); // Make c the size of a
99     if (N != b.N)
100         std::cerr << "vector::+ : cant add two vectors of different size!"
101                 << std::endl;
102     else
103         for (unsigned int i=0; i<N; i++)
104             c.entries[i] = entries[i] + b.entries[i];
105     return(c);
106 }

```

belongs to the Vector class.

set  $c = a + b$

(but a is implicit)

First thing to notice is that, although `+` is a binary operator, it seems to take only one argument. This is because, when we call the operator, `c = a + b` then `a` is passed **implicitly** to the function and `b` is passed **explicitly**. Therefore, for example, `a.N` is known to the function simply as `N`.

The temporary object `c` is used inside the object to store the result. It is this object that is returned. Neither `a` or `b` are modified.

**CS319 – Week 8**  
**Week 8: Linear Systems, and Operator Overloading**

**END OF PART Part 5**

CS319 – Week 8  
Week 8: Linear Systems, and Operator Overloading

Start of ...

**PART 6: The “pointy”, `this`, and `=` operators**

We now want to see another way of accessing the implicitly passed argument. First, though, we need to learn a little more about pointers, and introduce a new piece of C++ notation.

First, remember that if, for example, `x` is a `double` and `y` is a pointer to `double`, we can set `y=&x`. So now `y` stores the memory address of `x`. We then access the contents of that address using `*y`.

Now suppose that we have an object of type `Vector` called `v`, and a *pointer to vector*, `w`. That is, we have defined

```
Vector v;  
Vector *w;
```

So  $(*w) = v$

Then we can set `w=&v`. Now accessing the member `N` using `v.N`, will be the same as accessing it as `(*w).N`.

It is important to realise that `(*w).N` is **not** the same as `*w.N`.

However, C++ provides a new operator for this situation: `w->N`, which is equivalent to `(*w).N`.

When writing code for functions, and especially overloaded operators, it can be useful to **explicitly** access the implicitly passed object.

That is done using the `this` pointer, which is a pointer to the object itself.

.....

As we've just noted, since `this` is a pointer, its members are accessed using either `(*this).N` or `this->N`.

The following simple (but contrived ) demonstration shows why we might like to use the `this` pointer.

The (original) constructor for our `Vector` class took an `integer` argument called `Size`:

```
// Original version
Vector::Vector (unsigned int Size)
{
    N = Size;
    entries = new double[Size];
}
```

Suppose, for no good reason, we wanted to call the passed argument `N`: Then we would get the code below:

```
// Silly, broken version
Vector::Vector (unsigned int N)
{
    N = N;
    entries = new double[N];
}
```

Surprisingly, this will compile, but will give bizarre results (before crashing). To get around this we must distinguish between the local, passed, variable `N`, and the class member `N`.

This can be done by changing the offending line to:

```
this->N = N;
```



A much better use of this is for the case where a function must return the address of the argument that was passed to it. This is the case of the assignment operator.

See `Vector08.cpp` for more details

```

100 // Overload the = operator. (assignment, as in C = b)
Vector &Vector::operator=(const Vector &b)
102 {
    if (this == &b) // So here this is a pointer to C.
        return(*this); // Taking care for self-assignment

    delete [] entries; // In case memory was already allocated

    N = b.N;
108 entries = new double[b.N];
    for (unsigned int i=0; i<N; i++)
110     entries[i] = b.entries[i]; ← copy!
112     return(*this);
}

```

(Not to be confused with `==` equalite `C == b`),