Test Student **1718-MA284**
Assignment Assignment-1 due 10/06/2017 at 05:00pm BST

1. (1 point)

The sets A and B are such that $|A| = 8$, $|B| = 2$, and $|A \cap B| = 0$.

Then $|A \cup B|$ is _____, and $|A \setminus B|$ is _____.

The sets C and D are such that $|C| = 5$, $|D| = 5$, and $|C \cap D| = 3$.

Then $|C \cup D|$ is _____, and $|C \setminus D|$ is _____.

The sets E and F are such that $|E| = 12$, $|F| = 18$, and $|E \cup F| = 23$.

Then $|E \cap F|$ is _____, and $|E \setminus F|$ is _____.

2. (1 point)

The sets A and B are such that $|A| = 12$, $|B| = 9$.

The largest possible value of $|A \cup B|$ is _____.

The smallest possible value of $|A \cup B|$ is _____.

The largest possible value of $|A \cap B|$ is _____.

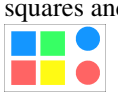
The smallest possible value of $|A \cap B|$ is _____.

The value of $|A \cap B| + |A \cup B|$ is _____.

3. (1 point) The power set of A is denoted $\mathcal{P}(A)$. Determine whether the given statement is true or false. Write T for true and F for false.

___1. $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$
___2. $\mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B)$
___3. $\mathcal{P}(A \setminus B) \subseteq \mathcal{P}(A) \setminus \mathcal{P}(B)$

4. (1 point) Michelle chooses one square and one circular disk from the
squares and disks shown below.



How many different ways can Michelle choose one square and one disk?

- A. 8
- B. 2
- C. 6
- D. 4

5. (1 point) A standard Missouri state license plate consists of a sequence of two letters, one digit, one letter, and one digit. How many such license plates can be made?

_____

A standard New York state license plate consists of a sequence of three letters followed by three digits. How many such license plates can be made?

_____

6. (1 point) A boy owns 2 pairs of pants, 7 shirts, 6 ties, and 6 jackets. How many different outfits can the boy wear to school if each outfit must consist of one of each item?

There are _____ different outfits.

7. (1 point) A bit is a 0 or a 1. A bit string of length 8 is a sequence of 8 digits, all of which are either 0 and 1.

(a) How many bit strings of length 8 are there?

_____

(b) How many bit strings of length 8 or less are there?
(Count the empty string of length zero also.)

_____

8. (1 point) This problem is taken from the delightful book "Problems for Mathematicians, Young and Old" by Paul R. Halmos.
Suppose that 971 tennis players want to play an elimination tournament. That means: they pair up, at random, for each round; if the number of players before the round begins is odd, one of them, chosen at random, sits out that round. The winners of each round, and the odd one who sat it out (if there was an odd one), play in the next round, till, finally, there is only one winner, the champion. What is the total number of matches to be played altogether, in all the rounds of the tournament?
Your answer: _____.
Hint: This is much simpler than you think. When you see the answer you will say "of course".

9. (1 point) 5 -letter "words" are formed using the letters A, B, C, D, E, F, G. How many such words are possible for each of the following conditions?
(a) No condition is imposed.
Your answer is : _____
(b) No letter can be repeated in a word.
Your answer is : _____
(c) Each word must begin with the letter A.
Your answer is : _____

(d) The letter C must be at the end.
Your answer is : _____
(e) The second letter must be a vowel.
Your answer is : _____

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10. (1 point) In how many ways can 3 different novels, 4 different mathematics books, and 1 biology book be arranged on a bookshelf if

(a) the books can be arranged in any order?
Answer: _____

(b) the mathematics books must be together and the novels must be together?
Answer: _____

(c) the novels must be together but the other books can be arranged in any order?
Answer: _____

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11. (1 point) A standard deck of cards consists of four suits (clubs, diamonds, hearts, and spades), with each suit containing 13 cards (ace, two through ten, jack, queen, and king) for a total of 52 cards in all.

How many cards in the deck are either a jack or a heart?
_____

How many cards are face cards or clubs?
_____

How many cards are red (diamonds or hearts) or queens?
_____

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12. (1 point) A survey of $1,000$ employees in a company revealed that 236 like rock music, 387 like pop music, 118 like jazz, 109 like pop and rock music, 53 like jazz and rock, 40 like pop and jazz, and 21 employees like all three.
How many employees do not like jazz, pop, or rock music?
_____
How many employees like pop but not jazz? _____

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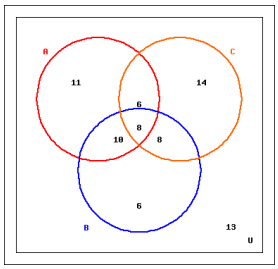
13. (1 point)



How many elements are contained in each of the following sets?

A : _____
B : _____
C : _____
U : _____
$A \cap (\overline{C \cup B})$: _____
$B \cap (\overline{A \cup C})$: _____

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14. (1 point) How many 4-element subsets containing the letter A can be formed from the set $\{A,B,C,D,E,F,G\}$?
Answer: _____

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15. (1 point) A DNA sequence can be represented as a string of the letters ACTG (short for adenine, cytosine, guanine, and thymine).
(a) How many DNA sequences are exactly 23 letters long?
_____
(b) Given a DNA sequence of length 23, how many single letter mutations are possible? _____
(c) Given a DNA sequence of length 23, how many double letter mutations are possible? _____

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16. (1 point) There are 9 portable mini suites (a.k.a. cages) in a row at the Paws and Claws Holiday Pet Resort. They are neatly labeled with their guests' names. There are 4 poodles and 5 tabbies. How many ways can the "suites" be arranged if:
a) there are no restrictions.
_____
b) cats and dogs must alternate.
_____
c) dogs must be next to each other.
_____
d) dogs must be next to each other and cats must be next to each other.
_____

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17. (1 point) A coin is tossed 13 times.
a) How many different outcomes are possible?
_____
b) How many different outcomes have exactly 4 heads?
_____
c) How many different outcomes have at least 2 heads ?
_____
d) How many different outcomes have at most 9 heads?
_____

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18. (1 point) You have 3 pairs of pants or skirts, 4 shirts or blouses, and 5 pairs of shoes. You can use them to wear _____ different outfits.
You are a participant in a peace conference with 10 participants. Everybody shakes everybody else's hand. There are _____ handshakes altogether.
A family of five is taking an extended vacation. Every day at lunch they stand in line at a cafeteria in a different order than ever before. On the last day, however, they can't help repeating a previous order. Their vacation lasted _____ days.

19. (1 point)

Recall that a *5-bit string* is a bit strings of length 5, and a bit string of weight 3, say, is one with exactly three 1's.

How many 5-bit strings are there? ____

How many 5-bit strings have weight 0? ____

How many 5-bit strings have weight 1? ____

How many 5-bit strings have weight 2? ____

How many 5-bit strings have weight 4? ____

How many 5-bit strings have weight 5? ____

How many 5-bit strings have weight 11? ____

20. (1 point)

How many 9-bit strings (that is, bit strings of length 9) are there which:

1. Start with the sub-string 101? ___
2. Have weight 5 (i.e., contain exactly five 1's) *and* start with the sub-string 101? ___
3. Either start with 101 *or* end with 11 (or both)? ___
4. Have weight 5 and either start with 101 or end with 11? ___