Test Student **1718-MA284**
Assignment Assignment-2 due 10/27/2017 at 05:01pm BST

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1. (1 point) A *ternary string* is a string made up of 0's, 1's. and 2's. How many ternary strings of length 7 are there?

Number of 7-digit ternary strings = _____

How many ternary strings of length 7 start with a 1?

Number of 7-digit ternary strings starting with 1 = _____

How many ternary strings of length 7 start with a 1 *and* end with a 1?

Number of 7-digit ternary strings starting and ending with 1 = _____

How many ternary strings of length 7 start with a 1 *or* end with a 1?

Number of 7-digit ternary strings starting or ending with with 1 = _____

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2. (1 point) Calculate each of the following. Your answer must be a number. No arithmetic operations are allowed in your answer. You can give your answer using scientific notation, i.e., writing 12345678 as 1.2345678E+07. Please give 7 places after your decimal point if you use scientific notation.

$\frac{400!}{40!360!} =$ _____.

$\frac{390!}{365!50!} =$ _____.

$\frac{670!-665!}{667!} =$ _____.

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3. (1 point) Simplify the expression

$$\frac{(2n+3)!}{(2n-3)!}.$$

$\frac{(2n+3)!}{(2n-3)!} =$ _____

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4. (1 point) How many different 9-letter permutations can be formed from 7 identical H's and two identical T's?

Answer: _____

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5. (1 point) How many three-letter "words" can be made from 4 letters "FGHI" if repetition of letters

(a) is allowed?

Your answer is : _____

(b) is not allowed?

Your answer is : _____

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6. (1 point) Five people applied for jobs at a store. Only two of these people will be hired.

The number of different pairs of people who could be hire is ___.

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7. (1 point) How many elements are in the union of four sets if each of the sets has 96 elements, each pair of sets share 52 elements, each triple of sets shares 30 elements and there are 7 elements in all four sets.

_____

8. (1 point)

After a round of "trick or treating", Candice has 8 KitKats and 12 Twix in her pillow case. Her mother asks her to share some (but not necessarily all) of the loot with her three younger brothers.

(A) How many different ways can she do this?

_____

(B) How many different ways can she do this if she gives at least one of each type of bar to each of her brothers?

_____

9. (1 point) How many ways are there to select 12 countries in the United Nations to serve on a council if 3 is selected from a block of 59, 2 are selected from a block of 64 and 7 are selected from the remaining 66 countries?

_____

10. (1 point)

Willie Wonka gives everyone who visits his factory 9 pieces of candy to take home. He never gives a person 2 or more pieces of the same type of candy. If Mr. Wonka has 26 different types of candy, in how many different ways could Mr. Wonka give a visitor his candy?

Mr. Wonka can distribute candy in _____ different ways.

If 180 people visit Mr. Wonka's factory each day, how many days could Mr. Wonka go without giving two visitors the same selection of candy

Mr. Wonka can go for _____ days without repeating candy selections.

11. (1 point)

A **multiset** is a collection of objects, just like a set, but can contain an object more than once (the order of the elements still doesn't matter). For example, $\{1,1,2,5,5,7\}$ is a multiset of size 6.

How many *sets* of size 4 can be made using the 10 digits: 0,1,...,9? ___.

How many *multisets* of size 4 can be made using the 10 digits: 0,1,...,9? ___.

12. (1 point)

How many non-negative integer solutions are there to the following **equation**?

$x_1 + x_2 + \cdots + x_9 = 7$

Answer: _____

How many non-negative integer solutions are there to the following **inequality**?

$x_1 + x_2 + \cdots + x_9 < 7$

Answer: _____

13. (1 point)

How many ways can you give 12 (identical) apples to your 5 favourite Mathematics lecturers (without any restrictions)?

Answer: _____

How many ways can you give 12 (identical) apples to your 5 favourite Mathematics lecturers if each of them gets at **least** one apple?

Answer: _____

How many ways can you give 12 (identical) apples to your 5 favourite Mathematics lecturers if each of them gets at **least two** apple?

Answer: _____

14. (1 point)

How many ways can you give 11 (identical) apples to your 7 favourite Mathematics lecturers (without any restrictions)?
Answer: _____
How many ways can you give 11 (identical) apples to your 7 favourite Mathematics lecturers if each of them gets at **least** one apple?
Answer: _____
How many ways can you give 11 (identical) apples to your 7 favourite Mathematics lecturers if each of them gets at **most** two apples?
Answer: _____

15. (1 point)

On Friday morning, before their Discrete Mathematics lecture, 6 students each leave one bag each in the Cloakroom.

How many ways can their bags be returned to them?
Answer: _____
How many ways can their bags be returned to them so that none of then get their own bags back?
Answer: _____
How many ways can their bags be returned to them so that **exactly one of them** gets their own bag back?
Answer: _____

16. (1 point)

How many "words" can you make from the letters MOY?
Answer: _____
How many "words" can you make from the letters NORE?
Answer: _____
How many "words" can you make from the letters CARRYDUFF?
Answer: _____

17. (1 point)

A Discrete Mathematics student comes from the village of KILLAHOURAGAIN. As an hilarious jape, on the way home from a late-night study session, he decides to rearrange the letters on one of the sign-posts.

How many arrangements of the letters in KILLAHOURAGAIN are possible?

Answer: _____
Of these arrangements, how many have all the L's together?
Answer: _____
Of these arrangements, how may have all the letters in alphabetical order?
Answer: _____

18. (1 point)

Our Indiscrete Mathematics course has

- 23 students from the the College of Arts, 13 of whom are female;
- 22 students from the the College of Engineering and Informatics, 6 of whom are female;
- 37 students from the the College of Science, 18 of whom are female.

How many ways can we choose a single class rep?
Answer: _____

How many ways can we choose **three reps**, one from each of the three Colleges?
Answer: _____
How many ways can we choose **three reps**, one from each of the three Colleges, so that **exactly one** is female?
Answer: _____

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19. (1 point) Are the following statements true or false?

? 1. $n! = (n-1)\big((n-1)! + (n-2)!\big)$ for $n \geq 2$
? 2. $\binom{n}{2} + \binom{n-2}{k-2} = \binom{n}{k} + \binom{k}{2}$
? 3. $\binom{n}{2}\binom{n-2}{k-2} = \binom{n}{k}\binom{k}{2}$
? 4. $\sum_{k=1}^{n} k(n+1-k) = \binom{n+3}{2}$
? 5. $\sum_{k=0}^{n} 2^k \binom{n}{k} = 3^n$
? 6. $\binom{n}{k} = \binom{n}{n-k}$
? 7. $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

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20. (1 point)

The University of Ballygobackwards has three colleges:

- the College of Science, which has 13 staff, 5 of whom are female;
- the College of Engineering, which has 8 staff, 2 of whom are female;
- and the College of Arts, which has 16 staff, 10 of whom are female.

Each college has a Dean, who is chosen from the staff in that college.

What is the total number of ways these **three** Deans can be chosen?
Answer: _____
How many ways can three Deans be chosen, so that at *least one* of them is female?
Answer: _____