Graph Theory Definitions

Graph: A collection of *vertices*, some of which are connected by *edges*. More precisely, a pair of sets V and E where V is a set of vertices and E is a set of 2-element subsets of V.

Adjacent: Two vertices are *adjacent* if they are connected by an edge. Two edges are *adjacent* if they share a vertex.

Bipartite graph: A graph for which it is possible to divide the vertices into two disjoint sets such that there are no edges between any two vertices in the same set.

Complete bipartite graph: A bipartite graph for which every vertex in the first set is adjacent to every vertex in the second set.

Complete graph: A graph in which every pair of vertices is adjacent.

Connected: A graph is *connected* if there is a path from any vertex to any other vertex.

Chromatic number: The minimum number of colors required in a proper vertex coloring of the graph.

Cycle: A path (see below) that starts and stops at the same vertex, but contains no other repeated vertices.

Degree of a vertex: The number of edges incident to a vertex.

Euler path: A path which uses each edge exactly once.

Euler circuit: An Euler path which starts and stops at the same vertex.

Multigraph: A *multigraph* is just like a graph but can contain multiple edges between two vertices as well as single edge loops (that is an edge from a vertex to itself).

Path: A sequence of vertices such that consecutive vertices (in the sequence) are adjacent (in the graph). A path in which no vertex is repeated is called *simple*.

Planar: A graph which can be drawn (in the plane) without any edges crossing.

Subgraph: We say that H is a *subgraph* of G if every vertex and edge of H is also a vertex or edge of G. We say H is an *induced* subgraph of G if every vertex of H is a vertex of G and each pair of vertices in H are adjacent in H if and only if they are adjacent in G.

Tree: A (connected) graph with no cycles. (A non-connected graph with no cycles is called a *forest*.) The vertices in a tree with degree 1 are called *leaves*.

Vertex coloring: An assignment of colors to each of the vertices of a graph. A vertex coloring is *proper* if adjacent vertices are always colored differently.