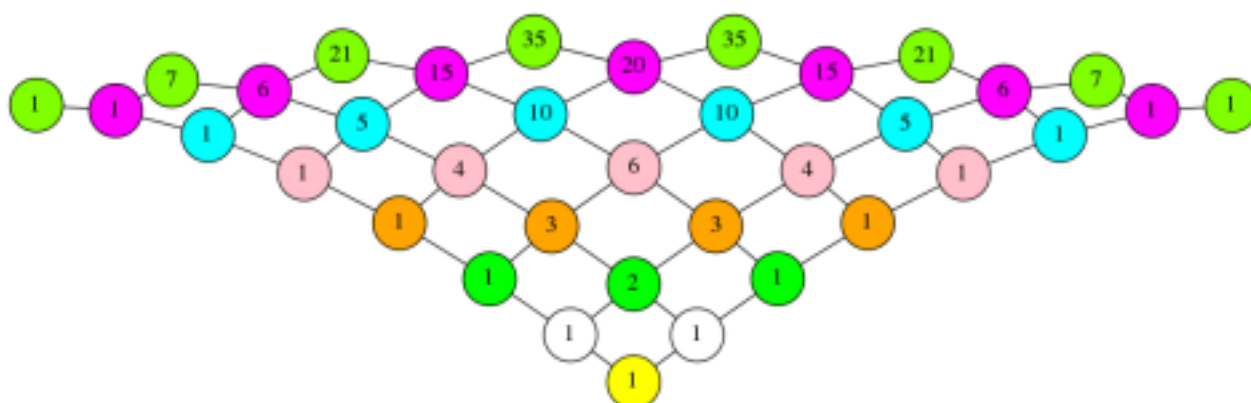


MA284 : Discrete Mathematics

**Week 1: Introduction to Discrete Mathematics; The Additive and Multiplicative Principles**

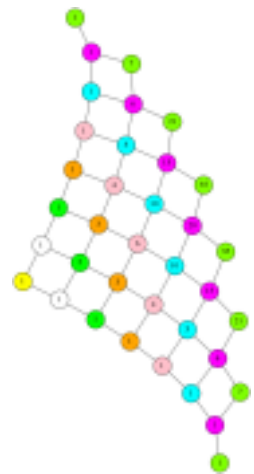
Dr Niall Madden

**6 & 8 September 2017**



Annotated slides from 6 September.

- 1 What is this module?
  - What/when/where
  - Tutorials
- 2 Textbook
- 3 What is Discrete Mathematics?
  - Problems in Combinatorics
  - Problems in graph theory
- 4 Mathematical Preliminaries
- 5 Why take 284???
- 6 Counting
- 7 Some examples
- 8 The Additive Principle
- 9 The Multiplicative Principle
- 10 Counting with Sets
- 11 Exercises



This is *Discrete Mathematics*: a mathematics module introduces the concepts of

- *enumerative combinatorics* (i.e., counting) and
- *graph theory* (i.e., the theory of graphs).

Don't worry: most of the rest of the definitions in this module will be more helpful than that!

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If you want to contact me, the best way is by email.

.....  
This module is taken by students in

- 2nd Science (Mathematics, Mathematical Science, Physics, E&O, Computer Science, Financial Mathematics and Economics, ...);
- 2nd Arts (Mathematics, Mathematics & Education, ...);
- **2nd Computer Science & IT (2BCT1);**
- **3rd Mathematics and Education;**
- Visiting student(s).

Given your *very* varied backgrounds, you will need to stay focused, and become practiced at communicating your own insights and challenges...

**Lectures:** Wednesday at 1pm in the Anderson theatre;  
Friday at 11am in AM200.

**Tutorials:** They will start in Week 3. **More details in a moment.**

**Web sites:** The online resource are at

- <http://www.maths.nuigalway.ie/~niall/MA284>  
There you'll find various pieces of information, including
  - these slides;
  - homework assignments,
- <http://NUIGalway.BlackBoard.com>
  - Announcements;
  - Grade centre;
  - etc...

**Work load:** 5 ECTS (60 is the typical total for a full-time programme)  
24 lectures, all in Semester 1  
Roughly 120 hours of student effort time.

**Lecture materials:** Slides for the week's classes will be available for download in advance of the Wednesday lecture.

These contain the main definitions, ideas, and examples (and some typos, probably!). And links.

They also contain exercises, which are of a similar style and standard as those on the final exam.

However, *these are not a complete record of the class.*

**Images:** Particularly in the second half of this course, there will be lots of pictures of graphs. These are mostly generated using

**Graphviz** <http://www.graphviz.org/> and/or NetworkX <https://networkx.github.io/>

I'll make the source code available. But if I forget, please ask!

**Assessment:** Your progress in, and commitment to, this course will be assessed as follows:

- **Continuous assessment:** There will be three online homework assignments, each worth 10%.
- **Final assessment:** There will be a 2 hour exam at the end of the semester, worth 70%.

**SUMS:** The School of Maths provides a free drop-in centre called

**SUMS: Support for Undergraduate Maths Students.**

SUMS opens from **2pm to 5pm, Monday to Friday**, from Monday, 19 September. For more information, see

<http://www.maths.nuigalway.ie/sums/>

**Devices:** The use of portable electronic devices during class is *encouraged*. For example, you might want to use it to check Wikipedia, or access the textbook.

*Be aware that these can be distracting to other students.*

*Please be considerate.*

**Other stuff:** Today is **Socs Day**! Why not (re)join the Mathematics Society? <https://www.facebook.com/MathsSocNUIG>

Also, consider joining our Student Chapter of SIAM:

<http://www.maths.nuigalway.ie/SIAM-Galway/>

Tutorials will start in Week 3 (week beginning 18 September).

You should attend *one tutorial per week*.

The times we used last year were:

	Mon	Tue	Wed	Thu	Fri
9 – 10					
10 – 11					
11 – 12			???		
12 – 1		???			
1 – 2					
2 – 3	???		???		
3 – 4	???				
4 – 5					
5 – 6					

If you would like a tutorial at a different time, please indicate your preferences by filling out the form at

<https://goo.gl/forms/Sk6S1glJqtJK4LM32> ← *Link!*

An extra tutorial time will be added



The main recommended text is

Oscar Levin, *Discrete Mathematics: an open introduction*, 2nd Edition. This is a free, open source textbook, available from <http://discretetext.oscarlevin.com>, in both printable and tablet/ereader-friendly versions. It is published under *Creative Commons Attribution-ShareAlike 4.0 International License*.

Other recommended texts include:

- Normal L Biggs, *Discrete Mathematics*, Oxford Science Publications.  
There are about 10 copies in the library at 510 BIG.
- Kenneth Rosen, *Discrete Mathematics and Its Applications*, McGraw-Hill.  
Located at 511 ROS.

Other books and resources will be mentioned as we go through the module.

Some related, fun, reading.

*Really Big Numbers*, by Richard Schwartz, published by the American Mathematical Society.

It is aimed a children, but is quite sophisticated. So you can learn some Discrete Mathematics while doing bed-time reading!

Watch at

<https://www.youtube.com/watch?v=cEOY9UAsCFM>



### **Four Colors Suffice: How the Map Problem Was Solved.**

Robin Wilson.

In the library at [511.5 WIL](#)

This is the story of the solution of one of most famous mathematical problems, that defied solution for nearly 150 years. It is also a treatise on what “proof” really means.

[Do you have any other suggestions?](#)

If calculus is “continuous mathematics”, then “discrete mathematics” is everything else! However, it is usually taken to include the following

- 1 Logic
- 2 Sets and set-theory;
- 3 Mathematics of Algorithms;
- 4 Recursion and induction;
- 5 Counting;
- 6 Discrete probability;
- 7 Graphs, trees and networks;
- 8 Boolean algebra;
- 9 Modelling computing (Turing machines and Finite State Machines).

But we will just focus on **counting (combinatorics)** and *graphs*.

### 1. Combinatorics.

How to count, the additive and multiplicative principles. The Binomial coefficients and some identities. The principal of Inclusion-Exclusion. Permutations and Combinations. Non-negative equations and inequalities. Derangements and distributions

### 2. Graph Theory.

Euler and the Koenigsberg Bridges Problem.

Eulerian and Hamiltonian graphs.

Tree graphs and bipartite graphs.

Planarity of Graphs.

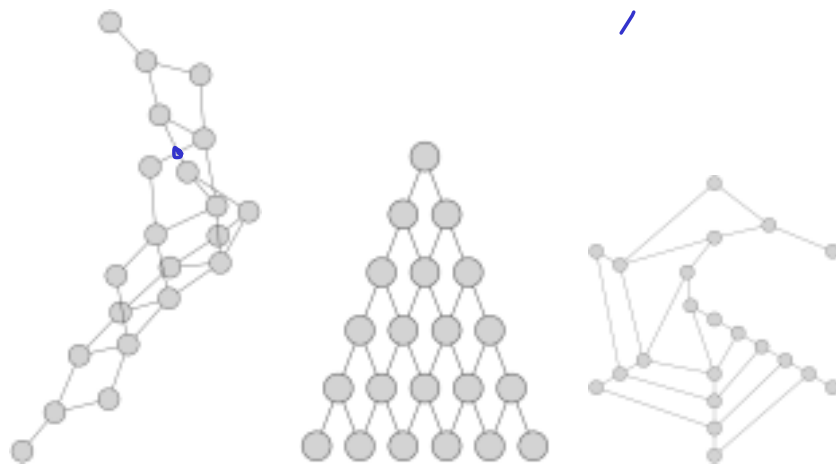
Eulers formula for a connected planar graph.

Planarity and the Platonic solids;

Colouring of Graphs.

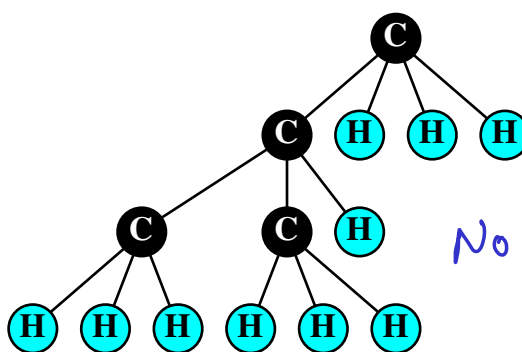
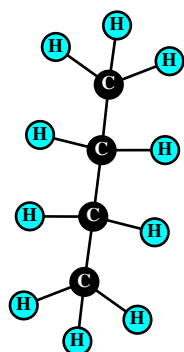


1. What are your chances of winning the Irish Lottery ("Lotto"). That is, what is the probability of correctly selecting **6** numbers from **47**?
2. If 500,000 people play the Lotto per week. What is the chance of a roll-over (i.e., nobody winning)?  
Last year, the Lotto changed from a **45** ball game to a **47** ball game. How have the chances of a roll-over changed?
3. For Wednesday's soccer match between Ireland and Serbia, a **23**-man squad was named.  
How many different ways are there of selecting the 11 starting players for the match?  
How many ways can one select (up to 3) of these players to be substituted during the game?
4. My password has 10 characters. Each character is an upper- or lower-case letter, or a digit. How long would it take you to crack my account?



1. Which of these graphs are the **same** (and what does that mean)?
2. Is it possible to draw all the graph on the left so that none of its edges intersect?
3. Can we colour the vertices so that no two adjacent vertices have the same colour?

4. Is there a “route” through the graph that visits every vertex once and only once?
5. How many regular polyhedra (platonic solids) are there?
6. Are all the graphs of *saturated hydrocarbon isomers* trees?



No loops.

There are very few prerequisites for this module. I will expect that

- you can reason logically;
- understand the concept of a *proof*, as know several proof techniques, such as *induction*.
- know what a matrix is, and how to multiply a matrix by a vector, and a matrix by a matrix.
- you are comfortable with the concept of **sets**, and the notation used to describe and manipulate them.
- you are comfortable with the concept of **functions**, and the notation used to describe and manipulate them.

### Exercise

Read Sections 0.3 (Sets) and 0.4 (Functions) in Chapter 0 of *Discrete Mathematics: an open introduction*



The most important reason for taking this module is that **Discrete mathematics is one of the most appealing, elegant, and applicable areas of mathematics.**

**Appealing:** The problems that we will consider are, I believe, easily motivated, but not trivial.

**Elegant:** The solutions to these problems involves some clever reasoning, but never tedious calculations.

**Applicable:** In spite of its classical origins, graph theory is one of the hottest topics in both pure and applied mathematics. For just one small example, read this article by Paddy Cosgrave, founder of the *Web Summit*, on *Maths and Conferences* ← *Link!*

Finished here 6/9/17