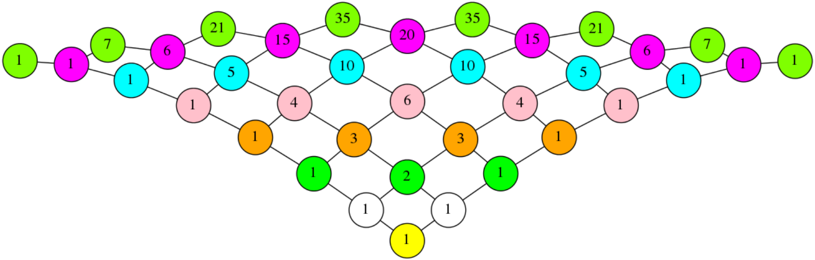
MA284 : Discrete Mathematics

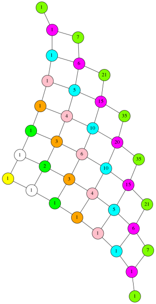
Week 1: Introduction to Discrete Mathematics; The Additive and Multiplicative Principles

Dr Niall Madden

6 & 8 September 2017



Outline



This is *Discrete Mathematics*: a mathematics module introduces the concepts of

- *enumerative combinatorics* (i.e., counting) and
- *graph theory* (i.e., the theory of graphs).

Don't worry: most of the rest of the definitions in this module will be more helpful than that!

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If you want to contact me, the best way is by email.

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This module is taken by students in

- 2nd Science (Mathematics, Mathematical Science, Physics, E&O, Computer Science, Financial Mathematics and Economics, ...);
- 2nd Arts (Mathematics, Mathematics & Education, ...);
- **2nd Computer Science & IT (2BCT1)**;
- **3rd Mathematics and Education**;
- Visiting student(s).

Given your *very* varied backgrounds, you will need to stay focused, and become practiced at communicating your own insights and challenges...

Lectures: Wednesday at 1pm in the Anderson theatre;
Friday at 11am in AM200.

Tutorials: They will start in Week 3. More details in a moment.

Web sites: The online resource are at

- http://www.maths.nuigalway.ie/∼niall/MA284
  There you'll find various pieces of information, including
  - these slides;
  - homework assignments,
- http://NUIGalway.BlackBoard.com
  - Announcements;
  - Grade centre;
  - etc...

Work load: 5 ECTS (60 is the typical total for a full-time programme)
24 lectures, all in Semester 1
Roughly 120 hours of student effort time.

Lecture materials: Slides for the week's classes will be available for download in advance of the Wednesday lecture.
These contain the main definitions, ideas, and examples (and some typos, probably!). And links.
The also contain exercises, which are of a similar style and standard as those on the final exam.
However, *these are not a complete record of the class*.

Images: Particularly in the second half of this course, there will be lots of pictures of graphs. These are mostly generated using **Graphviz** `http://www.graphviz.org/` and/or NetworkX `https://networkx.github.io/`
I'll make the source code available. But if I forget, please ask!

Assessment: Your progress in, and commitment to, this course will be assessed as follows:

- **Continuous assessment:** There will be three online homework assignments, each worth 10%.
- **Final assessment:** There will be a 2 hour exam at the end of the semester, worth 70%.

SUMS: The School of Maths provides a free drop-in centre called

SUMS: Support for Undergraduate Maths Students.

SUMS opens from **2pm to 5pm, Monday to Friday**, from Monday, 18 September. For more information, see http://www.maths.nuigalway.ie/sums/

Devices: The use of portable electronic devices during class is *encouraged*. For example, you might want to use it to check Wikipedia, or access the textbook.
Be aware that these can be distracting to other students. Please be considerate.

Other stuff: Today is **Socs Day**! Why not (re)join the Mathematics Society? https://www.facebook.com/MathsSocNUIG
Also, consider joining our Student Chapter of SIAM: http://www.maths.nuigalway.ie/SIAM-Galway/

Tutorials will start in Week 3 (week beginning 18 September).
You should attend *one tutorial per week*.

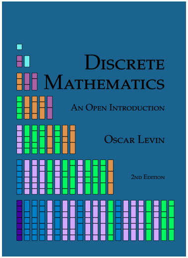
The times we used last year were:

	Mon	Tue	Wed	Thu	Fri
9 – 10					
10 – 11					
11 – 12			???		
12 – 1		???			
1 – 2					
2 – 3	???		???		
3 – 4	???				
4 – 5					
5 – 6					

If you would like a tutorial at a different time, please indicate your preferences by filling out the form at
`https://goo.gl/forms/Sk6S1glJqtJK4LM32` ← *Link!*

The main recommended text is

Oscar Levin, Discrete Mathematics: an open introduction, 2nd Edition. This is a free, open source textbook, available from http://discretetext.oscarlevin.com, in both printable and tablet/ereader-friendly versions. It is published under Creative Commons Attribution-ShareAlike 4.0 International License.



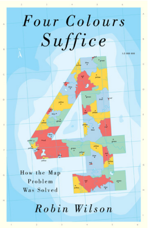
Other recommended texts include:

- Normal L Biggs, *Discrete Mathematics*, Oxford Science Publications. There are about 10 copies in the library at `510 BIG`.
- Kenneth Rosen, *Discrete Mathematics and Its Applications*, McGraw-Hill. Located at `511 ROS`.

Other books and resources will be mentioned as we go through the module.

Some related, fun, reading.

Really Big Numbers, by Richard Schwartz, published by the American Mathematical Society.
It is aimed a children, but is quite sophisticated. So you can learn some Discrete Mathematics while doing bed-time reading!
Watch at
https://www.youtube.com/watch?v=cEOY9UAsCFM



Four Colors Suffice: How the Map Problem Was Solved.
Robin Wilson.
In the library at 511.5 WIL
This is the story of the solution of one of most famous mathematical problems, that defied solution for nearly 150 years. It is also a treatise on what "proof" really means.

Do you have any other suggestions?

If calculus is "continuous mathematics", then "discrete mathematics" is everything else! However, it is usually taken to include the following

1. Logic
2. Sets and set-theory;
3. Mathematics of Algorithms;
4. Recursion and induction;
5. Counting;
6. Discrete probability;
7. Graphs, trees and networks;
8. Boolean algebra;
9. Modelling computing (Turing machines and Finite State Machines).

But we will just focus on **counting (combinatorics)** and *graphs*.

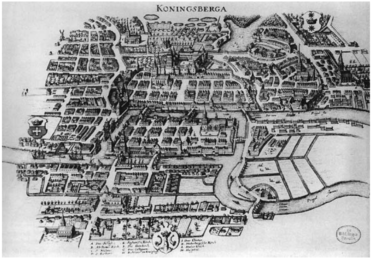
What is Discrete Mathematics?

1. Combinatorics.

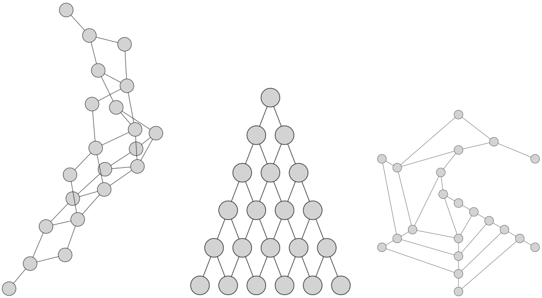
How to count, the additive and multiplicative principles. The Binomial coefficients and some identities. The principal of Inclusion-Exclusion. Permutations and Combinations. Non-negative equations and inequalities. Derangements and distributions

2. Graph Theory.

Euler and the Koenigsberg Bridges Problem.
Eulerian and Hamiltonian graphs.
Tree graphs and bipartite graphs.
Planarity of Graphs.
Eulers formula for a connected planar graph.
Planarity and the Platonic solids;
Colouring of Graphs.



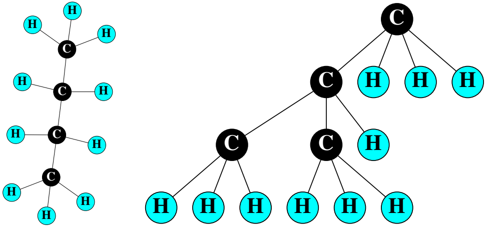
1. What are your chances of winning the Irish Lottery ("Lotto"). That is, what is the probability of correcting selecting **6** numbers from **47**?
2. If 500,000 people play the Lotto per week. What is the chance of a roll-over (i.e., nobody winning)?
   Last year, the Lotto changed from a **45** ball game to a **47** ball game. How have the chances of a roll-over changed?
3. For Wednesday's soccer match between Ireland and Serbia, a **23**-man squad was named.
   How many different ways are there of selecting the 11 starting players for the match?
   How many ways can one select (up to 3) of these players to be substituted during the game?
4. My password has 10 characters. Each character is an upper- or lower-case letter, or a digit. How long would it take you to crack my account?



1. Which of these graphs are the **same** (and what does that mean)?
2. Is it possible to draw all the graph on the left so that none of its edges intersect?
3. Can we colour the vertices so that no two adjacent vertices have the same colour?

4. Is there a "route" through the graph that visits every vertex once and only once?
5. How may regular polyhedra (platonic solids) are there?

6. Are all the graphs of *saturated hydrocarbon isomers* trees?



There are very few prerequisites for this module. I will expect that

- you can reason logically;
- understand the concept of a *proof*, as know several proof techniques, such as *induction*.
- know what a matrix is, and how to multiply a matrix by a vector, and a matrix by a matrix.
- you are comfortable with the concept of **sets**, and the notation used to describe and manipulate them.
- you are comfortable with the concept of **functions**, and the notation used to describe and manipulate them.

Exercise

Read Sections 0.3 (Sets) and 0.4 (Functions) in Chapter 0 of *Discrete Mathematics: an open introduction*

The most important reason for taking this module is that **Discrete mathematics is one of the most appealing, elegant, and applicable areas of mathematics**.

Appealing: The problems that we will consider are, I believe, easily motivated, but not trivial.

Elegant: The solutions to these problems involves some clever reasoning, but never tedious calculations.

Applicable: In spite of its classical origins, graph theory is one of the hottest topics in both pure and applied mathematics. For just one small example, read this article by Paddy Cosgrave, founder of the *Web* Summit, on *Maths and Conferences* $\leftarrow$ *Link!*

Combinatorics is the mathematics of *counting*. It has its origins in the 17^{th} century, when a systematic study of gambling began.

The simplest method of counting is *simple enumeration* $=$ *"Point and count"*.

1. How many students from `2BA` are in this room?

2. How many anagrams are there of the letters `NUI`?

   Usually we don't want to make a list of all possibilities:

3. How many car licence plates are there of the form `XXX-yyy`, where X is a letter and y is a digit?
   Answer: There are 17,576,000, but we don't want to list them all.

The first techniques that we will study for doing are called

The Additive and Multiplicative Principles

For more information see, in order of importance:

1. Chapter 1 (Counting) of Oscar Levin's *Discrete Mathematics: an open introduction*.
2. Chapter 4 of Rosen's *Discrete Mathematics and Its Applications* (`511 ROS`).
3. Chapter 3 of Biggs' *Discrete Mathematics* (`510 BIG`).

1. There are 5 starters and 6 main-courses on a restaurant's menu. How many choices do you have if
   (a) You only want one dish;
   (b) You would like a starter and a main-course?

2. A standard deck of cards has 26 red cards, and 12 face/court cards.
   (a) How many ways can you select a card that is red *and* face card?
   (b) How many ways can you select a card that is red *or* face card?

Think about these questions as we go through the following examples...

Example

The NUIG Animal Shelter has 4 cats and 6 dogs in need of a home. You would like a new pet (but just one!). How many choices do you have?

The Additive Principle

If event A can occur m ways, and event B can occur n (disjoint) ways, then event "A **or** B" can occur in $m + n$ ways.

Example

1. Can we use the additive principle to determine how many two letter "words" **begin with** either A **or** B?
2. Can we use the additive principle to determine how many two letter "words" **contain** either A **or** B?

The Additive Principle

If event A can occur m ways, and event B can occur n (disjoint) ways, then event "A **or** B" can occur in $m + n$ ways.

Example

The NUIG Animal Shelter has 4 cats, 6 dogs, and 7 donkeys in need of a home. How many choices do you have for a new pet?

The Additive Principle

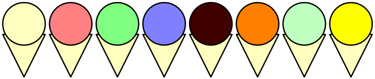
If event A can occur m ways, and event B can occur n **disjoint** ways, then event "A **or** B" can occur in $m + n$ ways.

Example

A deck of cards has 26 red cards and 12 "face"-cards.

1. How many ways can you pick a red card?
2. How many ways can you pick a face-card?
3. How many ways can you pick a card that is red *or* is a face-card?

This last example is important because it emphasizes the importance of the sets being **disjoint**.



Example

Your favourite ice-cream shop has **8** flavours of ice-cream.
You can also choose between a cone, a waffle, and a cup.
How many choices to you have?

The Multiplicative Principle

If event A can occur m ways, and each possibility allows for B to occur in n (disjoint) ways, then event "A **and** B" can occur in $m \times n$ ways.

Example

The NUIG Animal Shelter has 4 cats and 6 dogs in need of a home. How many choices do you have if you would like a cat and a dog as pets?

Example

The NUIG Animal Shelter also has 7 donkeys. How many choices to you have if you want a cat, a dog and a donkey?

A set is a collection of things. The items in a set are called *elements*.
Examples:

- The set of natural numbers from 1 to 10 is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- The set of upper-case letters is $\{A, B, \ldots, Y, Z\}$
- The set of students registered for Discrete Mathematics has 190 elements.
- A set is *unordered*.

You should be familiar with the following basic elements of set notation:

$$\{\cdot\} \quad \in \quad \notin \quad \subseteq \quad \cup \quad \cap \quad \emptyset \quad |\cdot| \quad \setminus$$

Example

Let $A = \{1,2,3\}$, $B = \{1,3,5\}$, and $C = \{2,4\}$.

- $2 \in A$, $\quad 4 \notin A$
- $\{1,3\} \subseteq A$
- $A \cup B = \{1,2,3,5\}$
- $A \cap B = \{1,3\}$,
  $B \cap C = \emptyset$,
- $|A| = 3$,
  $|B \cap C| = 0$,
- $A \setminus B = \{2\}$
  $A \setminus C = \{1,3\}$

Also, for any set X, $\quad X \subseteq X \quad \emptyset \subseteq X$.

Here are a set of exercises to help you work through the material presented during this week's classes.

All but the last are taken either directly from the textbook, or with minor edits.

You do not have to submit your solutions to be graded.

Q0. Download a copy of the course textbook, Levin's *Discrete Mathematics: an open introduction* from http://discretetext.oscarlevin.com. Read Chapter 0, and then...

(a) Do Exercises 1, 2, 3 and 4 on Page 12, of Chapter 0.

(b) Find out what the *Power Set* is. Do Exercises 9–16 on P13.

(c) After doing the above exercises, compare your answers with those provided in Appendix B of the textbook.

Q1. Your wardrobe consists of 5 shirts, 3 pairs of pants, and 17 bow ties. How many different outfits can you make?

Q2. For your job interview at the NUIG Animal Shelter, you must wear a tie. You own 3 regular (boring) ties and 5 (cool) bow ties. How many choices do you have for your neck-wear?

Q3. You realize that the interview is actually for ClownSoc, so you should probably wear both a regular tie and a bow tie. How many choices do you have now?

Q4. Your DVD collection consists of 9 comedies and 7 horror movies. Give an example of a question for which the answer is:

(a) 16.
(b) 63.

Q5. If $|A| = 10$ and $|B| = 15$, what is the largest possible value for $|A \cap B|$? What is the smallest? What are the possible values for $|A \cup B|$?

Q6. If $|A| = 8$ and $|B| = 5$, what is $|A \cup B| + |A \cap B|$?