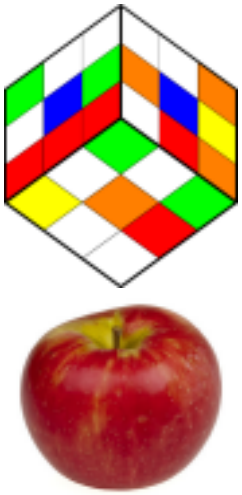
Annotated slides from Wednesday

MA204/MA284 : Discrete Mathematics

Week 5: Stars and Bars

http://www.maths.nuigalway.ie/~niall/MA284/

4 and 6 October, 2017



These slides are based on Sections 1.4 and 1.5 of Oscar Levin's *Discrete Mathematics: an open introduction*.

Announcements

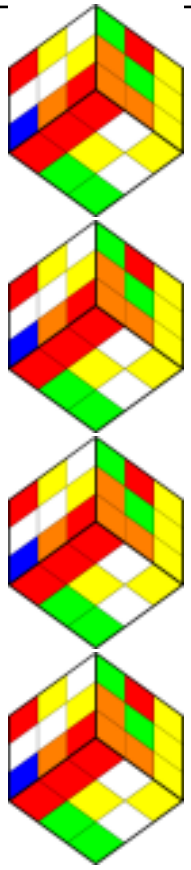
1. Assignment 1 is due 5pm, Friday 6 Oct 2017
   To access the assignment, go to
   http://mathswork.nuigalway.ie/webwork2/1718-MA284

2. Friday's lecture will take place in the Tyndall lecture theatre (because AM200 will be in use for Open Day).

Suppose you have some number of identical Rubik's cubes to distribute to your friends. Start by creating a single row of the cubes. Now find the number of different ways you can distribute the cubes provided:

1. you have 3 cubes to give to 2 people.
2. you have 4 cubes to give to 2 people.
3. you have 5 cubes to give to 2 people.
4. you have 3 cubes to give to 3 people.
5. you have 4 cubes to give to 3 people.
6. you have 5 cubes to give to 3 people.

Make a conjecture about how many different ways you could distribute 7 cubes to 4 people. Explain.
What if each person were required to get *at least one* cube? How would your answers change?



Think about this question during this lecture...

Every day you give some apples to your lecturers. Today you have **7** apples. How many ways can you give them to the **4** lecturers you have today?

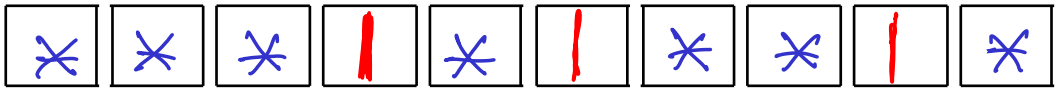
GE	NS	NM	JH	
3	1	2	1	= 7
2	2	1	2	= 7
0	7	0	0	= 7.

Every day you give some apples to your lecturers. Today you have **7** apples. How many ways can you give them to the **4** lecturers you have today?

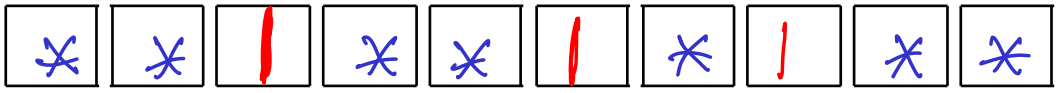
One can represent any solution by filling out 10 boxes with 7 stars and 3 bars.

Examples:

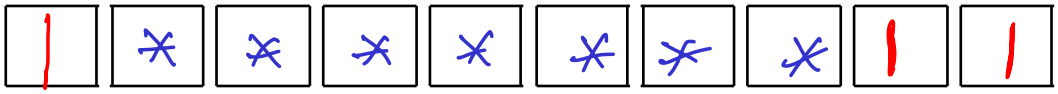
1. GE = 3 NS NM JH



2. GE = 2 NS = 2 NM - JH



3. GE = 0 NS = 7 NM = 0 JH = 0.



Every day you give some apples to your lecturers. Today you have **7** apples. How many ways can you give them to the **4** lecturers you have today?

- *Every solution can be represented by 10 boxes, each with a star or a bar.*
- There are 7 stars and 3 bars in total.
- We can choose any 3 of the 10 boxes in which to place the bars, and then put the stars in the rest.
- **So we have** $\binom{10}{3}$ **choices for where to put the bars**.

$$\binom{10}{3} = \frac{10!}{3!\,7!} = \binom{10}{7}$$

So choosing 3 boxes for bars is the same as choosing 7 boxes for stars.

Definition (Multiset)

A *multiset* is a set of objects, where each object can appear more than once. As with an ordinary set, order does not matter.

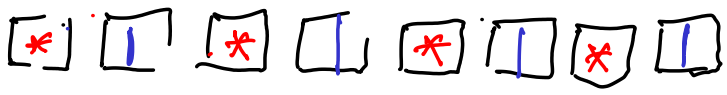
Examples: The sets $\{a, b, c\}$ and $\{a, b, c, a\}$ are the same

The multisets $\{a, b, c\}$ and $\{a, b, c, a\}$ are different.

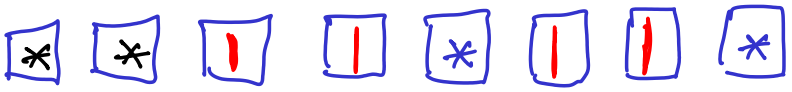
But $\{a, b, c, a\}$ is the same as $\{a, a, b, c\}$, (order does not matter).

Each solution to the "7 apples & 4 lecturers" problem is a multiset: elements are lecturers and the number of times they occur is the number of apples they get

How many *multisets* of size 4 can you form using numbers $\{1, 2, 3, 4, 5\}$?

Eg $\{1,2,3,4\}$ = 

$\{1,2,2,5\}$.

$\{1,1,3,5\}$ 

So we need 8 boxes with 4 stars & 4 bars.

Answer: $\binom{8}{4}$.

How many *multisets* of size n can you form using the numbers $\{1, 2, 3, , \ldots, k\}$?

This corresponds to giving n apples to k people.

Each solution can be represented with

n stars and $k-1$ bars

So there are $\binom{n+k-1}{k-1} = \binom{n+k-1}{n}$ solutions.

Example

MA204 Semester 1 Examination 2014/15: Q2(a)

1. In how many ways can one distribute **ten** €1 coins to four students?
2. In how many ways can one distribute **ten** €1 coins to four students so that each student receives at least €1?

Q1: We want to distribute $n=10$ identical objects among $k=4$ distinct people. This can be done with $n=10$ stars and $k-1=3$ bars.

ANS: $\binom{13}{3} = \binom{13}{10} = 286 = \binom{n+k-1}{k-1}$

Q2. To answer this, first give each €1, then distribute the remaining $n=6$ coins in $\binom{n+k-1}{k-1} = \binom{9}{3} = 84$.