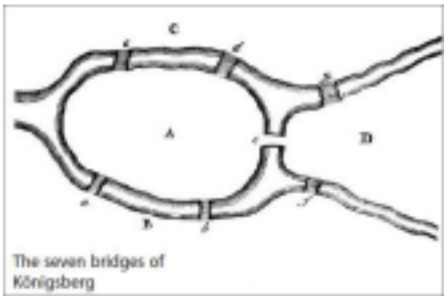
MA284 : Discrete Mathematics

Week 7: Introduction to Graph Theory.

http://www.maths.nuigalway.ie/~niall/MA284/

18 and 20 October, 2017



See also §1.6 and §4.0 of Levin's *Discrete Mathematics: an open introduction*.

Assignment 2 is open

To access the assignment, go to
http://mathswork.nuigalway.ie/webwork2/1718-MA284

There are **20** questions. You may attempt each one up to **10** times.
This assignment contributes 10% to your final grade for Discrete Mathematics.

Deadline: 5pm, Friday, 27 October.

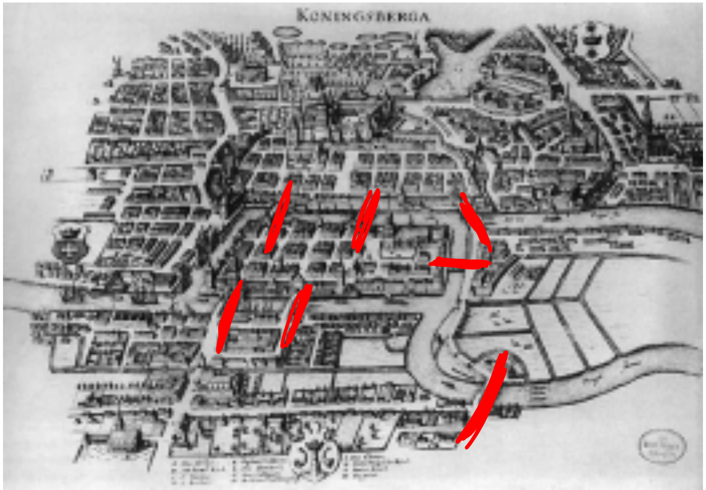
. . . .

Please complete the online survey on MA284 to provide us with feedback on improving it. The link is in an announcement on Blackboard.

Graph Theory is a branch of mathematics that is several hundred years old. Many of its discoveries were motivated by practical problems, such as determining the smallest number of colours needed to colour a map.

It is unusual in that its beginnings can be traced to a precise date.

Königsberg in Prussia (now Kaliningrad, Russia) had seven bridges. Is it possible it walk through the town in such a way that you cross each bridge once and only once?



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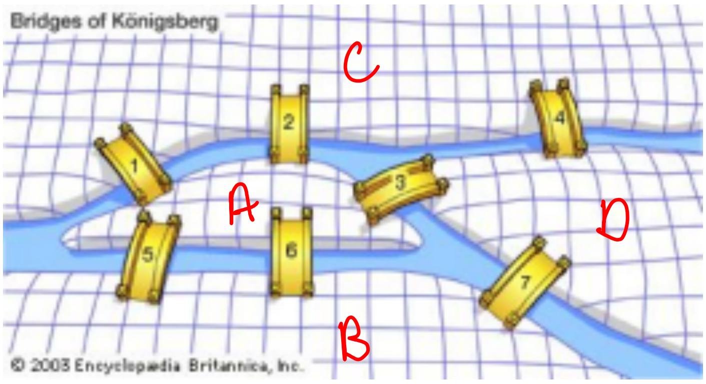

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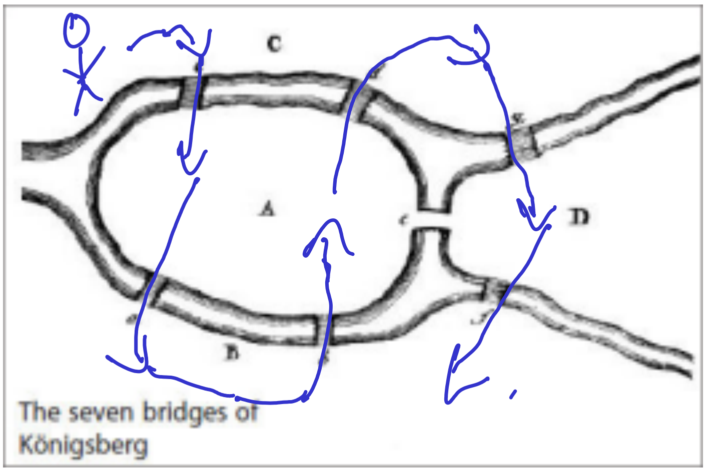
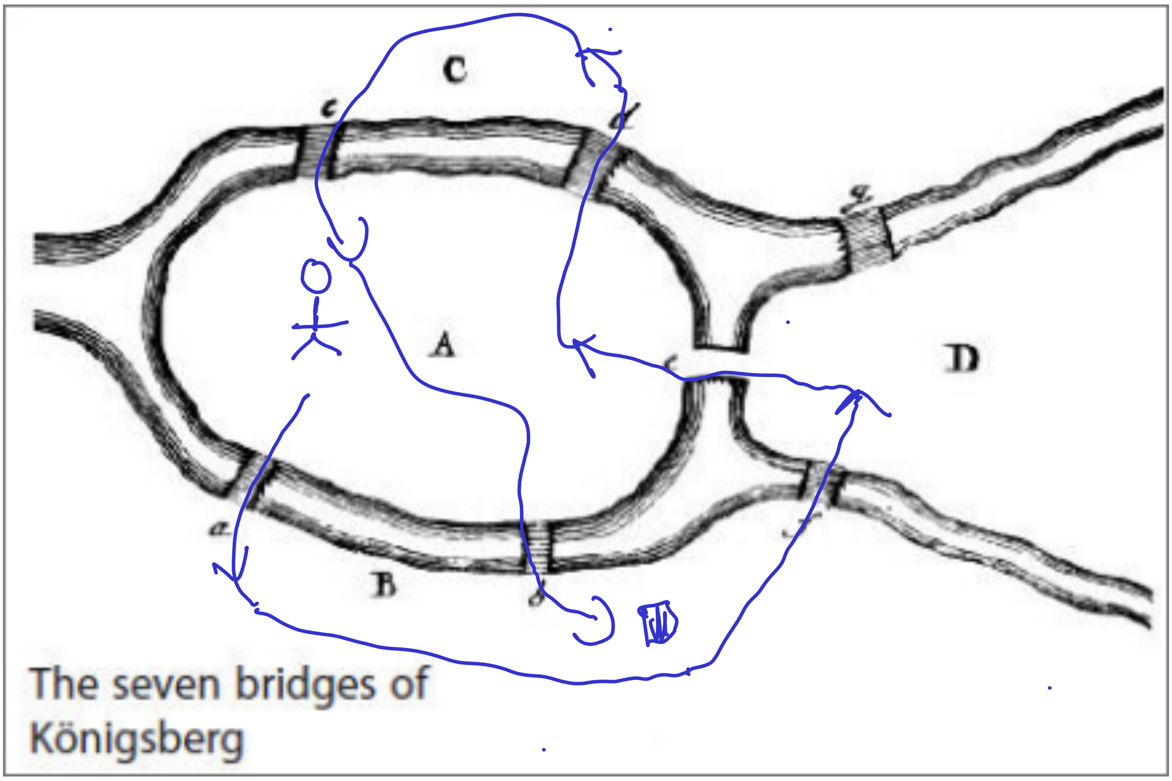


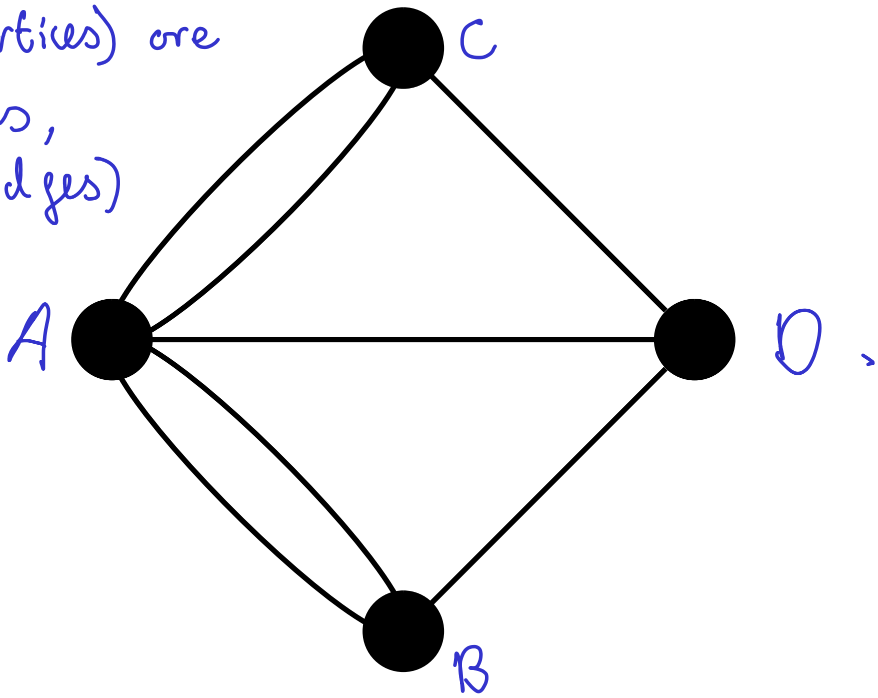
The seven bridges of Königsberg

Is it possible it walk through the town in such a way that you cross each bridge once and only once?



Here is another way of stating the same problem. Consider the following picture, which shows 4 dots connected by some lines.

The "dots" (vertices) are land regions, and lines (edges) are bridges



Is it possible to trace over each line once and only once (without lifting up your pencil)? You must start and end on one of the dots.

 "Undirected Graph" $\Leftrightarrow \{a, d\} = \{d, a\}$

Graph

A *GRAPH* is a collection of

- "vertices" (or "nodes"), which are the "dots" in the above diagram.
- "edges" joining pair of vertices.

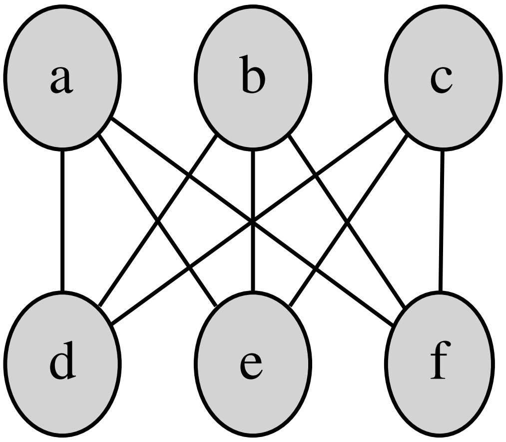
If the graph is called G (say), we often define it in terms of its *edge set* and *vertex set*. That is we write

E

V

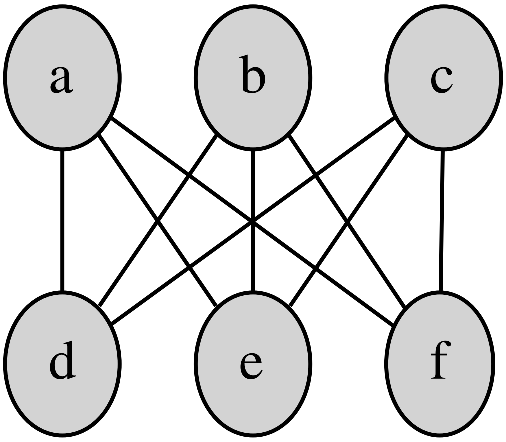
$$G = (V, E),$$

where V is the set of vertices and E is the set of edges.



$V = \{a, b, c, d, e, f\}$

$E = \{ \{a, d\}, \{a, e\}, \{a, f\}, \{b, d\}, \{b, e\}, \{b, f\}, \{c, d\}, \{c, e\}, \{c, f\} \}$



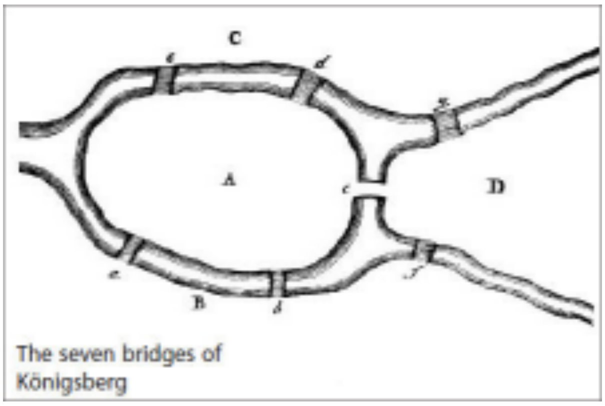
If two vertices are connected by an edge, we say they are *adjacent*.

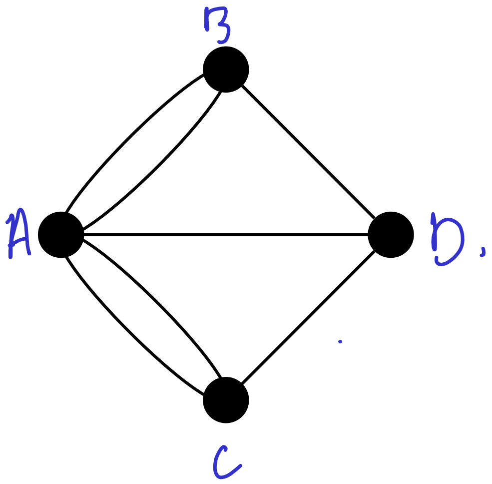
Here a is adjacent to d, e & f
but not to b & c.

So "x is adjacent to y" if $\{x,y\} \in E$.

Graphs are used to represent collections of objects where there is a special relationship between certain pairs of objects.

For example, in the Königsberg problem, the land-masses are vertices, and the edges are bridges.



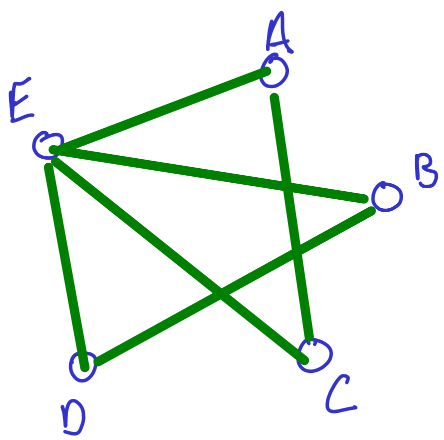


(Example 4.0.1 of the text-book)

Aoife, Brian, Conor, David and Edel are students in a *Indiscrete Mathematics* module.

- Aoife and Conor worked together on their assignment.
- Brian and David also worked together on their assignment.
- Edel helped everyone with their assignments.

Represent this situation with a graph.

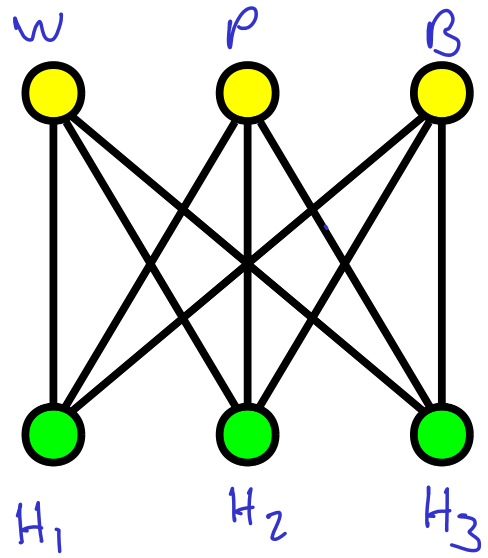


$V = \{A, B, C, D, E\}$

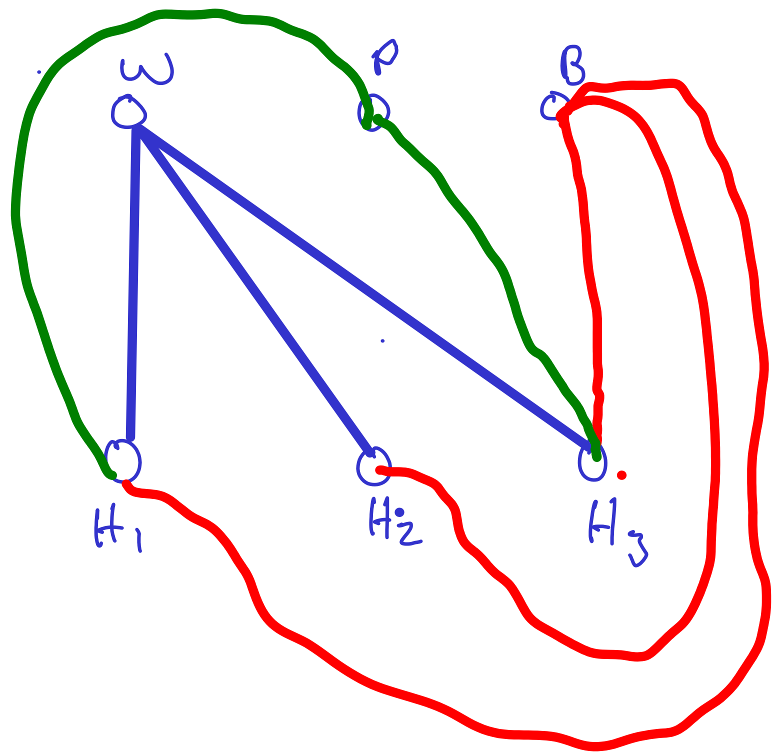
$E = \{ \{A, C\}, \{B, D\}, \{E A\}, \{E B\}, \{E, C\}, \{E D\} \}.$

The Three Utilities Problem; also Example 4.0.2 in the text-book.

We must make water, power and broadband connections to three houses.
Is it possible to do this without the conduits crossing?

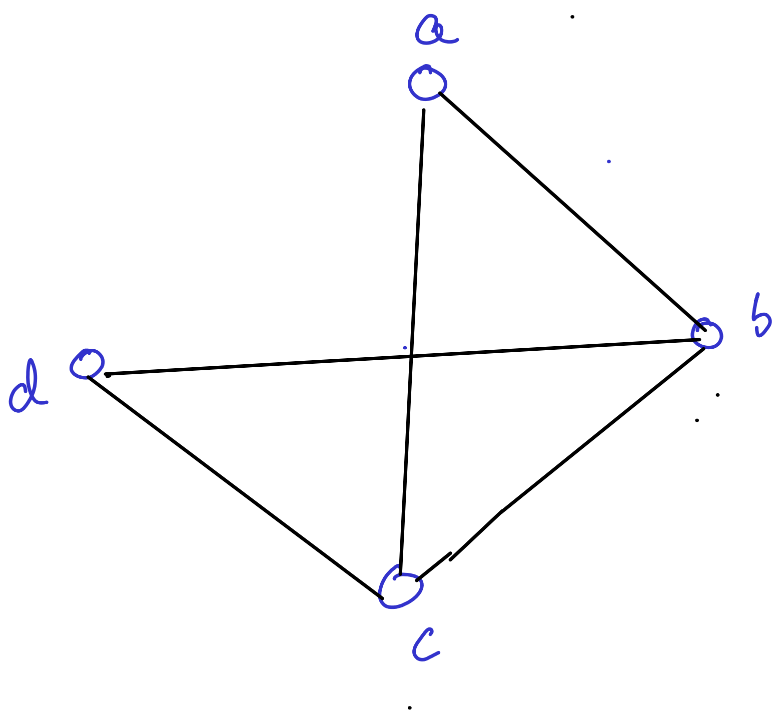


" Graph is not planar".

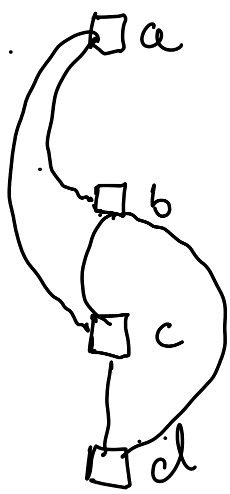


Sketch the graph with Vertex Set $V = \{a, b, c, d\}$ and Edge Set

$$E = \{\{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}\}$$



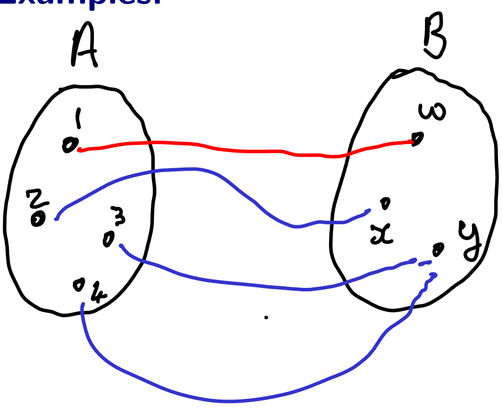
This is not the only way to sketch this graph.



Recall the $f : A \to B$ is a *function* that maps every elements of the set A onto some element of set B. (We call A the "domain", and B the "codomain".)
Each element of A gets mapped to exactly one element of B.

If $f(a) = b$ where $a \in A$ and $b \in B$, we say that "the image of a is b".
Or, equivalently, "b is the image of a".

Examples:



$f(1) = w$
$f(2) = x$
$f(3) = y$
$f(4) = y$.

This is a function.

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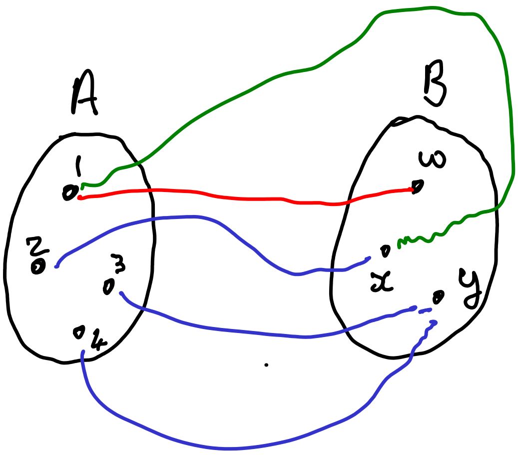

$f(1) = w$
$f(2) = x$
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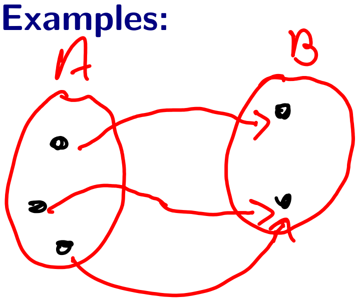
$f(4) = y$.
$f(1) = x$

This is NOT a function. because f sends 1 to 2 different elements of B.

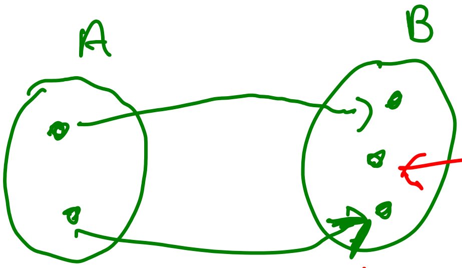
Finished here Wednesday.

When every element of B is the image of some element of A, we say that the function is *SURJECTIVE* (also called "onto").

Examples:



Surjective.



not the image of anything: So f is not surjective