MA284 : Discrete Mathematics

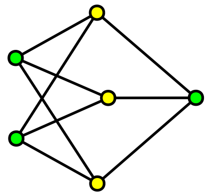
Week 8: The fundamentals of graph theory;
Planar Graphs
25 and 27 October, 2017

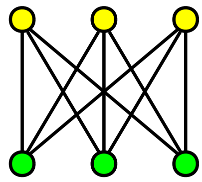
1 Definitions
- 1. A graph
- 2. Paths and connected graphs
- 3. Complete graphs
- 4. Vertex degree
- 5. Bipartite graphs
- 6. Subgraphs
- 7. Named graphs

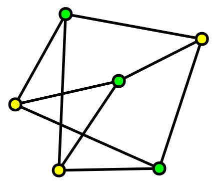
2 Planar graphs
- Faces, edges and vertices
- Euler's formula

3 Non-planar graphs

4 Exercises







See also Sections 4.1 and 4.2 of Levin's *Discrete Mathematics: an open introduction*.

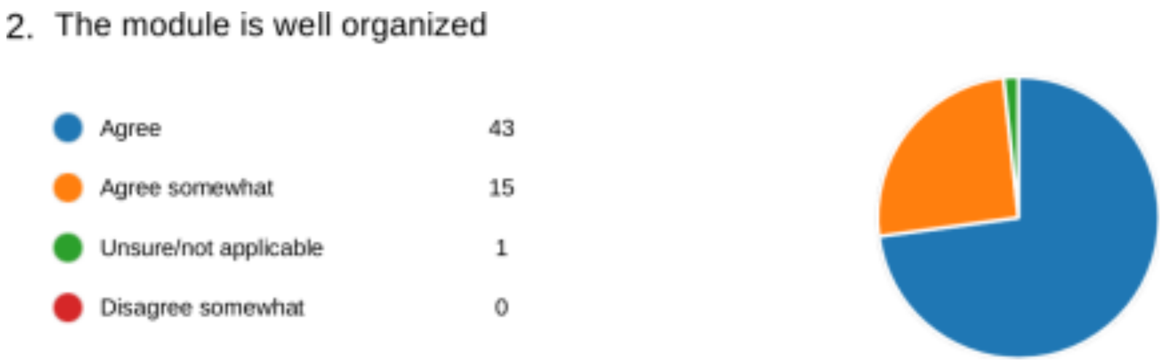
ASSIGNMENT 2 IS OPEN

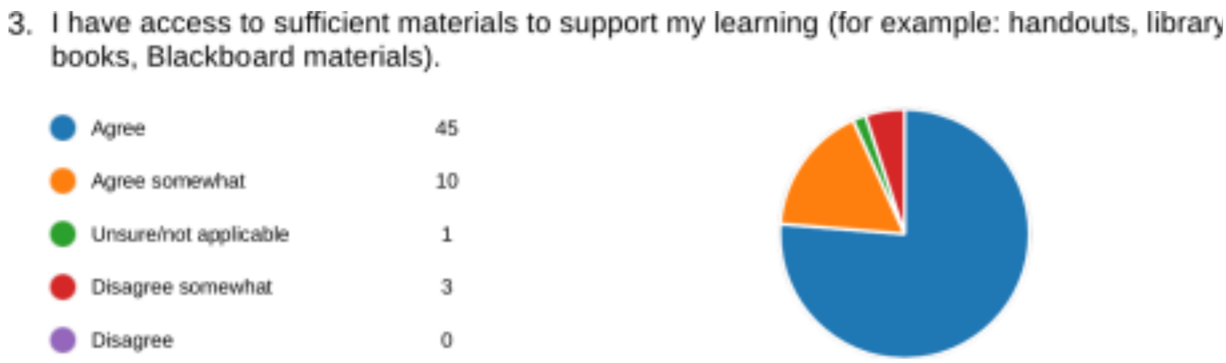
To access the assignment, go to
http://mathswork.nuigalway.ie/webwork2/1718-MA284

There are **20** questions. You may attempt each one up to **10** times.
This assignment contributes 10% to your final grade for Discrete Mathematics.

Deadline: 5pm, Friday, 27 October, 2017.

Thank you for your feedback on Discrete Mathematics. There were 56 responses. Here are a few:





Feedback on MA284

4. The feedback I have received is helping me improve my learning

Agree	19
Agree somewhat	15
Unsure/not applicable	17
Disagree	6
Disagree somewhat	2

5. The lectures are clear and well prepared.

Agree	48
Agree somewhat	9
Unsure/not applicable	2
Disagree somewhat	0
Disagree	0

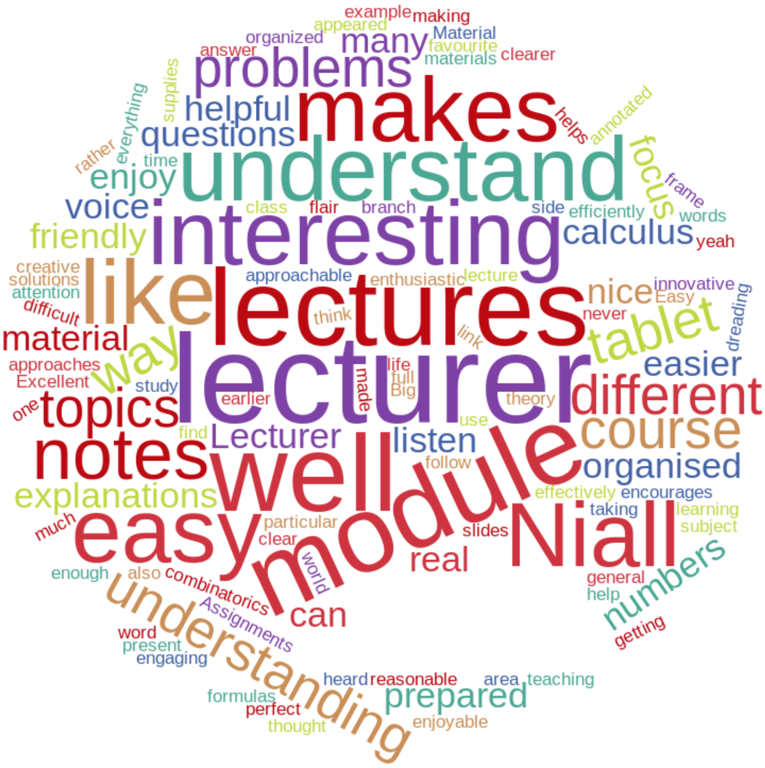
6. The lecturer(s) is/are helpful, approachable, and effective.

Agree	46
Agree somewhat	11
Disagree somewhat	2
Unsure/not applicable	0
Disagree	0

7. The tutorials/labs are helpful and effective.

Agree	29
Agree somewhat	10
Unsure/not applicable	15
Disagree somewhat	4
Disagree	1

10. What do you like about this module?



What suggestions can you offer that would help make this module a more valuable learning experience for you?

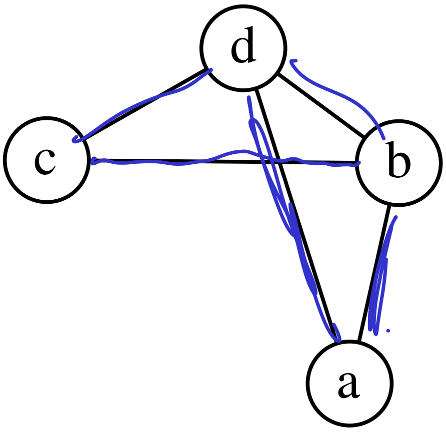
- More examples in lectures
- Module is too lecture based... students should feel free to shout questions out.
- I'd like to move faster but I know people might get left behind
- Less questions but more challanging ones would be great.
- More example of the type of questions that will appear on the exams
- More descriptive notes.
- Non-graded home work as well as the usual graded homework...
- Offer service similiar to clickers or something to make lecture more engaging
- The overhead lecture notes to be uploaded onto blackboard that are written during class,
- Shorter web works
- The tutorial should be improved

Recall from last week... a graph

A *GRAPH* is a collection of **vertices**, and **edges** joining pair of vertices.
A graph is defined in terms of its *edge set* and *vertex set*. That, the graph G with vertex set V and edge set E is written as

$$G = (V, E).$$

If two vertices are connected by an edge, we say they are *adjacent*.



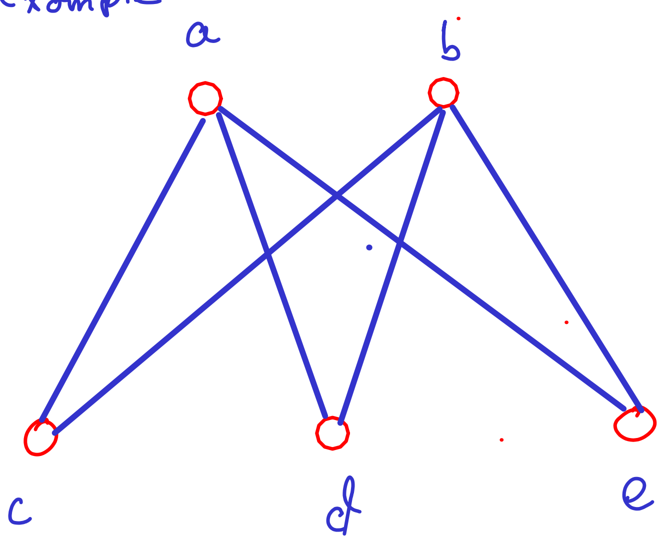
$V = \{a, b, c, d\}$

$E = \{\{a,b\}, \{b,c\}, \{d,a\}, \{d,b\}, \{c,d\}\}$

Here a is adjacent to b & d.

A **PATH** is a sequence of adjacent vertices in a graph. We often will talk about a *path* between two vertices. And, since there can be more than one, the *SHORTEST PATH* is particularly important.

Example



Paths from a to e:

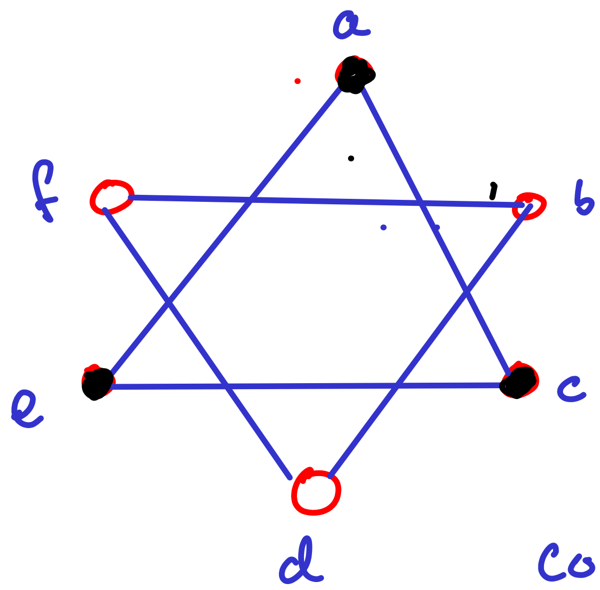
{a, c, b, d, a, e}

or

{a, d, b, e}

or {a; e}, which is the shortest.

A graph is **CONNECTED** if there is a path between every pair of vertices.



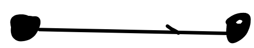
This is not connected: there is no path from a to b (for example).

It has 2 connected components.

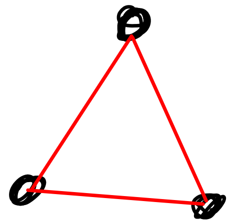
A graph is **COMPLETE** if every pair of vertices are adjacent. This family of graphs is VERY important. They are denoted K_n – the complete graph on n vertices.

K_1

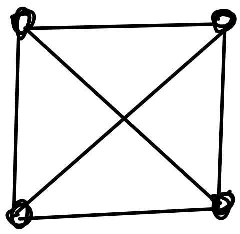
K_2



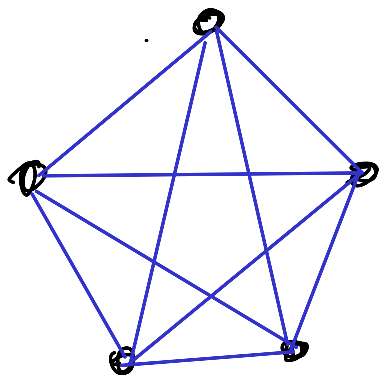
K_3



K_4

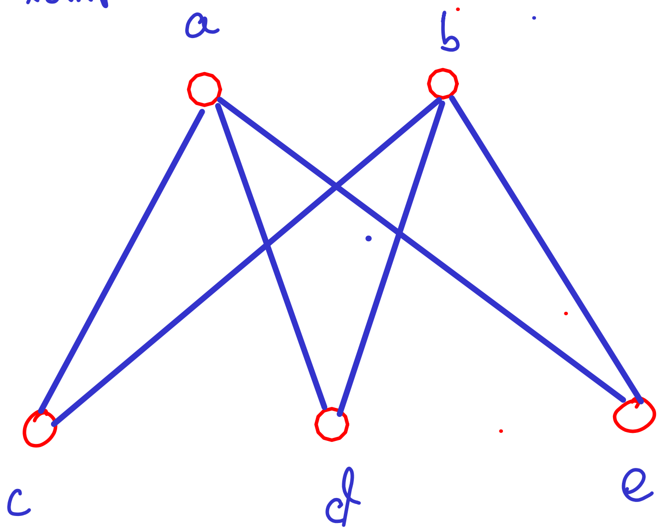


K_5



The **DEGREE** of a vertex is the number of edges emanating from it.

Example



a and b have degree 3

c, d, e have degree 2.

we write $deg(a) = 3$

$deg(e) = 2$

Degree table

Vertex	degree
a	3
b	3
c	2
d	2
e	2
Total:	12.

If we know the degree of every vertex in the graph then we know the number of edges.

Every edge contributes to the degree of two vertices.

So, the number of edges is half the degree sum.

Also, the degree sum is always even.

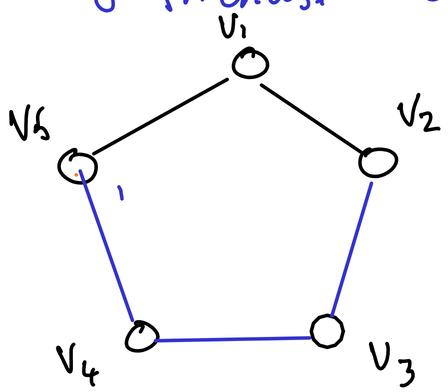
Example (MA284 Semester 1 Exam, 2015/2016, Q3(a))

Among a group of five people,

(i) is it possible for everyone to be friends with exactly <u>two</u> of the other people in the group?

(ii) is it possible for everyone to be friends with exactly <u>three</u> of the other people in the group?

(i) If we draw this as a graph, there are 5 vertices (= people), with an edge between every pair of friends. So every vertex has degree 2.



Example (MA284 Semester 1 Exam, 2015/2016, Q3(a))

Among a group of five people,

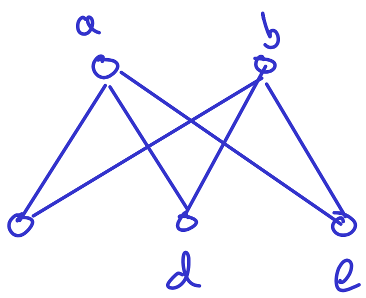
(i) is it possible for everyone to be friends with exactly <u>two</u> of the other people in the group?

(ii) is it possible for everyone to be friends with exactly <u>three</u> of the other people in the group?

(ii) If this could happen, we would have a graph with 5 vertices, all of degree 3. So the degree sum would be 15. But this is not possible, since the degree sum must be even.

If it is possible to divide the vertex set, V, into two disjoint sets, V_1 and V_2, such that there are no edges between any two vertices in the same set, then the graph is **BIPARTITE**.

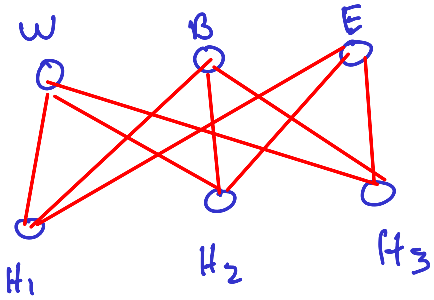
Eg



$V_1 = \{a, b\}$, $V_2 = \{c, d, e\}$

Finished here Wednesday

Eg (Utility Graph)



$V_1 = \{W, B, E\}$

$V_2 = \{H_1, H_2, H_3\}$